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Connected circular open subsets of $\mathbb{C}^n$

biholomorphisms from a bounded connected open circular

(H Cartan) Let $0 \in \Omega_1, \Omega_2 \subseteq \mathbb{C}^n$ be connected circular open subsets, Ω_1 is bounded and

$$F : \Omega_1 \to \Omega_2, F(0) = 0$$

is a biholomorphism. Then F is a linear transformation.

We have

$$\begin{aligned} F^{-1}(F(z)) &= z \\ \implies \mathfrak{D}_0(F^{-1}) \circ \mathfrak{D}_0 F &= \mathsf{Id} \\ \implies \mathfrak{D}_0(F^{-1}) &= (\mathfrak{D}_0 F)^{-1} \end{aligned}$$

- Now consider the holomorphic map

  $$\begin{aligned} H_\theta &: \Omega_1 \to \Omega_1 \\ H_\theta(z) &:= F^{-1}(\exp(-i\theta)F(\exp(i\theta)z)) \end{aligned}$$

  which satisfies

  $$H(0) = 0, \mathfrak{D}_0 H = \mathsf{Id}$$

- By

  (H Cartan) Let $\Omega \subseteq \mathbb{C}^n$ be a connected bounded open subset and

  $$F : \Omega \to \Omega$$

  be holomorphic. If for some $p \in \Omega$ we have $F(p) = p, \mathfrak{D}_p F = \mathsf{Id}$, then $F \equiv \mathsf{Id}_\Omega$.

  $H \equiv \mathsf{Id}$.

- So

  $$F(\exp(i\theta)z) = \exp(i\theta)F(z)$$

for every $\theta \in \mathbb{R}, z \in \Omega_1$.

- Thus F is linear.

Current note has 0 direct children and 0 total descendants.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - Cn conn open circular Connected circular open subsets of $\mathbb{C}^n$

And it has 36 siblings.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - 1Hol Holomorphic functions on spaces over $\mathbb{C}$ of dimension 1
    - circle packing Circle packing on $\mathbb{R}^2$
    - circle packing to Riemann map Circle packing converges to the Riemann biholomorphism
    - Cn conn open bounded Bounded connected open subsets of $\mathbb{C}^n$
    - Cn conn open circular Connected circular open subsets of $\mathbb{C}^n$
    - cont Continuous functions on $\mathbb{R}^d$
    - cube dyadic Dyadic cubes
    - curves Curves
    - derivative Differentiable functions
    - forms Differential forms on $\mathbb{R}^n$
    - Fourier-Wigner Fourier-Wigner transform
    - harmonic composed conformal Harmonic functions composed with conformal maps
    - Hilbert Hilbert transform
    - hol harmonic disk-circle Fourier-Cauchy-Poisson correspondence of holomorphic and harmonic functions on the unit disk and their boundary values
    - Hol sets Holomorphic subsets of $\mathbb{C}^n$
    - hypersurf 2n reg Regular hypersurfaces in $\mathbb{R}^{2n}$
    - hypersurf or Orientable hypersurfaces in $\mathbb{R}^n$
    - KG

$$\partial_t^2 + \sum_{i=1}^{n} v_i^2 \partial_{x_i}^2 + m^2$$

    - Laplace Laplace operator on $\mathbb{R}^n$

- Lmeas Lebesgue measurable subsets of and functions on $\mathbb{R}^n, T^n, S^n$
- Lmeas bd of open Lebesgue measure of boundary of open sets in $\mathbb{R}^n$
- met density Metric density of subsets of $\mathbb{R}^n$
- Mobius n-sphere Mobius endomorphisms
- monotone Monotone functions on $\mathbb{R}$
- periodic int Cauchy Cauchy integral of periodic functions
- poly int Polygons with integer vertices
- R 2 open smooth End Open smooth maps $U \subseteq \mathbb{R}^2 \to \mathbb{C}$
- R n discrete subg Discrete subgroups of $\mathbb{R}^n$
- R n discrete subg cocpt Discrete cocompact subgroups of $\mathbb{R}^n$, flat tori
- RC ramified germs Ramified germs of smooth and holomorphic functions
- Rn open Open subsets of $\mathbb{R}^n$
- Rn open Riem Open subsets of $\mathbb{R}^n$ equipped with the flat metric
- smooth quasi-analytic Quasi-analytic smooth functions on $\mathbb{R}$
- star shaped Star-shaped subsets of $\mathbb{R}^n$
- Vec ODEs in $\mathbb{R}^n \leftrightarrow$ Vector fields in $\mathbb{R}^n$
- wave

$$\partial_t^2 + \sum_{i=1}^{n} v_i^2 \partial_{x_i}^2$$