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$\widehat{\ } : L^2(0,\infty) \cong_{\text{Hilb}} \mathscr{O}^2(H^2_{\mathrm{U}})$

Definition.

$$\mathscr{O}^p(H^2_{\mathrm{U}}) := \left\{ f \in \mathscr{O} \,\middle|\, \sup_{y\in(0,\infty)} \int_{\mathbb{R}+iy} |f|^p < \infty \right\}$$

[1]

Let $f \in L^2(0,\infty) \le L^2(\mathbb{R})$ and $z \in H^2_{\mathrm{U}}$ consider

$$|f(t)\exp(itz)| = \left|f(t)e^{-t\Im(z)}\right|$$
$$\implies t \mapsto f(t)\exp(itz) \in L^2(0,\infty)$$

so we have the Fourier transform

$$\hat{f}(z) := \int_{(0,\infty)} f(t)\exp(itz)$$

for all $z \in H^2_{\mathrm{U}}$.

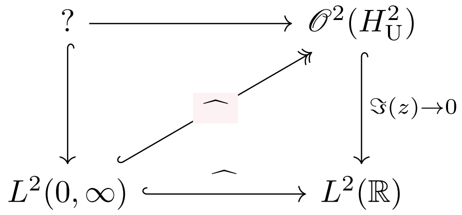
(Paley-Wiener)

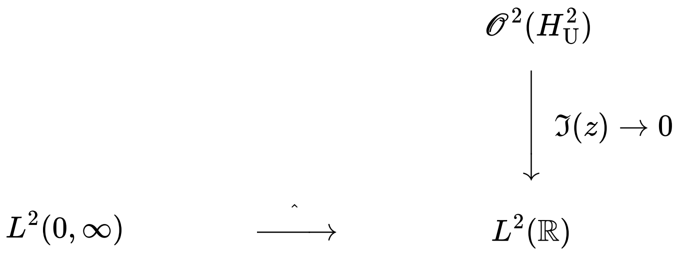
$$\hat{\ }: L^2(0,\infty) \cong_{\text{Hilb}} \mathscr{O}^2(H^2_{\mathrm{U}})$$

[2]

For any $\delta > 0$ and $y \ge \delta$ we have

$$\left|\int_{x\in(0,\infty)} f(x)e^{2\pi ixz}\right| \le \int_{x\in(0,\infty)} |f(x)|e^{2\pi x|z|}$$
$$\le \sqrt{\int_{(0,\infty)} |f|^2}\sqrt{\int_{x\in(0,\infty)} e^{-4\pi x\delta}}$$





Current note has 0 direct children and 0 total descendants.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - Fourier
      - L2 Rpos to Hardy2 upper 1 $\widehat{\ } : L^2(0,\infty) \cong_{\mathsf{Hilb}} \mathscr{O}^2(H^2_{\mathrm{U}})$

And it has 10 siblings.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - Fourier
      - L2 bdint Fourier transform on $L^2(-A,A) \leq L^2(\mathbb{R})$
      - L2 Rpos to Hardy2 upper 1 $\widehat{\ } : L^2(0,\infty) \cong_{\mathsf{Hilb}} \mathscr{O}^2(H^2_{\mathrm{U}})$
      - Rn Fourier transform on $\mathbb{R}^n$
      - S1 Fourier transform on S^1, Fourier series on $[0,1]$
      - S1 abs Functions on S^1 with absolutely converging Fourier series, $\check{l}^1(S^1)$
      - S1 dist Fourier transform of distributions on S^1

- S1 L1toC0 $\widehat{\ }: L^1[0,1] \to \mathcal{C}_0(\mathbb{Z},\mathbb{C})$
- S1 L2tol2 $\widehat{\ }: L^2[0,1] \cong_{\mathsf{Hilb}} l^2(\mathbb{Z},\mathbb{C})$
- subsets Fourier transform of measurable subsets
- unsharp Unsharpness principles

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1. book.pdf ↩
2. Definition.

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