

Info

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Hilbert transform

Let $\psi \in \mathcal{S}(\mathbb{R}^d)$ and consider the integral



$$\int_{[-\epsilon, \epsilon]} \frac{\psi(x)}{x} dx$$

The function $\frac{1}{x}$ is not integrable on $[-\epsilon, \epsilon]$. As ψ is bounded on $[-\epsilon, \epsilon]$,

$$\frac{M_0}{|x|} \leq \frac{|\psi(x)|}{|x|}$$

so $\frac{\psi(x)}{x}$ is also not integrable on $[-\epsilon, \epsilon]$.

However, as $\frac{1}{x}$ is odd, we have

$$\int_{\epsilon \leq |x| \leq 1} \frac{\psi(x)}{x} dx = \int_{\epsilon \leq |x| \leq 1} \frac{\psi(x) - \psi(0)}{x} dx$$

and by [mean value theorem](#) we have

$$\left| \frac{\psi(x) - \psi(0)}{x} \right| \leq \|\psi'\|_\infty$$

Thus,

$$0 \leq \left| \int_{\epsilon \leq |x| \leq 1} \frac{\psi(x)}{x} dx \right| \leq \|\psi'\|_\infty (2\epsilon)$$

so the limit exists

$$\lim_{\epsilon \rightarrow 0} \int_{\epsilon \leq |x| \leq 1} \frac{\psi(x)}{x} dx$$

Definition.

$$\mathcal{S}(\mathbb{R}^d) \rightarrow \mathbb{C}$$
$$\psi \mapsto \lim_{\epsilon \rightarrow 0} \int_{\epsilon \leq |x| \leq 1} \frac{\psi(x)}{x} + \int_{|x| \geq 1} \frac{\psi(x)}{x}$$

Current note has 0 direct children and 0 total descendants.

- [stamp](#) stamp
 - [Rf](#) subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
 - [Hilbert](#) Hilbert transform

And it has 36 siblings.

- [stamp](#) stamp
 - [Rf](#) subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
 - [1Hol](#) Holomorphic functions on spaces over \mathbb{C} of dimension 1
 - [circle packing](#) Circle packing on \mathbb{R}^2
 - [circle packing to Riemann map](#) Circle packing converges to the Riemann biholomorphism
 - [Cn conn open bounded](#) Bounded connected open subsets of \mathbb{C}^n
 - [Cn conn open circular](#) Connected circular open subsets of \mathbb{C}^n
 - [cont](#) Continuous functions on \mathbb{R}^d
 - [cube dyadic](#) Dyadic cubes
 - [curves](#) Curves
 - [derivative](#) Differentiable functions
 - [forms](#) Differential forms on \mathbb{R}^n
 - [Fourier-Wigner](#) Fourier-Wigner transform
 - [harmonic composed conformal](#) Harmonic functions composed with conformal maps
 - [Hilbert](#) Hilbert transform
 - [hol harmonic disk-circle](#) Fourier-Cauchy-Poisson correspondence of holomorphic and harmonic functions on the unit disk and their boundary values
 - [Hol sets](#) Holomorphic subsets of \mathbb{C}^n
 - [hypersurf 2n reg](#) Regular hypersurfaces in \mathbb{R}^{2n}
 - [hypersurf or](#) Orientable hypersurfaces in \mathbb{R}^n

- [KG](#)

$$\partial_t^2 + \sum_{i=1}^n v_i^2 \partial_{x_i}^2 + m^2$$

- [Laplace](#) Laplace operator on \mathbb{R}^n
- [Lmeas](#) Lebesgue measurable subsets of and functions on \mathbb{R}^n, T^n, S^n
- [Lmeas bd of open](#) Lebesgue measure of boundary of open sets in \mathbb{R}^n
- [met density](#) Metric density of subsets of \mathbb{R}^n
- [Mobius n-sphere](#) Mobius endomorphisms
- [monotone](#) Monotone functions on \mathbb{R}
- [periodic int Cauchy](#) Cauchy integral of periodic functions
- [poly int](#) Polygons with integer vertices
- [R 2 open smooth End](#) Open smooth maps $U \subseteq \mathbb{R}^2 \rightarrow \mathbb{C}$
- [R n discrete subg](#) Discrete subgroups of \mathbb{R}^n
- [R n discrete subg cocpt](#) Discrete cocompact subgroups of \mathbb{R}^n , flat tori
- [RC ramified germs](#) Ramified germs of smooth and holomorphic functions
- [Rn open](#) Open subsets of \mathbb{R}^n
- [Rn open Riem](#) Open subsets of \mathbb{R}^n equipped with the flat metric
- [smooth quasi-analytic](#) Quasi-analytic smooth functions on \mathbb{R}
- [star shaped](#) Star-shaped subsets of \mathbb{R}^n
- [Vec](#) ODEs in $\mathbb{R}^n \leftrightarrow$ Vector fields in \mathbb{R}^n
- [wave](#)

$$\partial_t^2 + \sum_{i=1}^n v_i^2 \partial_{x_i}^2$$