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Holomorphic subsets in $\mathbb{C}^n$

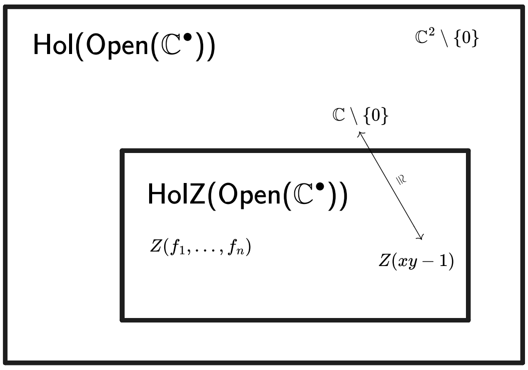
Definition. Definition

Let $U \subseteq \mathbb{C}^n$ be an open subset. A **holomorphic zero set** of U is a zero set of finitely many holomorphic functions $f_1, \dots f_n \in \mathscr{O}(U)$

$$Z(f_1, \dots, f_n) := \{z \in U | f_1(z) = f_2(z) = \cdots = f_n(z) = 0\} = \bigcap_i Z(f_i)$$

A **holomorphic subvariety** of U is a subset $X \subseteq U$ such that for each $p \in X$ there is a nb V_p in U such that $V_p \cap X$ is a holomorphic subset of U, that is,

$$\exists f_1, \dots f_n \in \mathscr{O}(V_p), V_p \cap X = Z(f_1, \dots, f_n)$$



> Let $X \subseteq U(\text{open}) \subseteq \mathbb{C}^n$ be a holomorphic subvariety.

Definition. Germs of holomorphic subvarieties

Let $X_1, X_2 \subseteq U$ be two holomorphic subvarieties of $U \subseteq \mathbb{C}^n$. Then X_1 and X_2 are **equivalent** at $p \in X_1 \cap X_2$ if there is a nb $U_p \subseteq U$ of p such that

$$X_1 \cap U_p = X_2 \cap U_p$$

Therefore, we have the surjection from the set $\mathsf{Hol}(U)$ of all holomorphic subvarieties of U onto the same quotiented out by *equivalence at p*

$$\mathsf{Hol}(U) \twoheadrightarrow \mathsf{Hol}(U)_p$$

A **germ** of holomorphic subvarieties of U at p is an element of $\mathsf{Hol}(U)_p$.

Definition. **Holomorphic functions on a subvariety**

Let $X \subseteq U(\text{open}) \subseteq \mathbb{C}^n$ be a holomorphic subvariety.

Then a function

$$f : X \to \mathbb{C}$$

is **holomorphic** if for each $p \in X$ there is a onb $U_p \subseteq \mathbb{C}^n$ such that f extends to $f \in \mathscr{O}(U_p)$.

Let $\mathbb{C}$-algebra of holomorphic functions on X be the set of all holomorphic functions on X

$$\mathscr{O}(X)$$

with pointwise operations and the sheaf of holomorphic functions on X be

$$\mathscr{O}_X : \mathsf{Open}(X) \to \mathsf{Alg}_{\mathbb{C}}$$
$$V \mapsto \mathscr{O}_X(V) := \mathscr{O}(V \cap X)$$

The stalk of the sheaf $\mathscr{O}_X$ at $p \in X$, that is, $\mathbb{C}$-algebra of germs of holomorphic functions

$$\mathscr{O}_{X,p}$$

is the **local ring** of X at p.

irreducible subvarieties

Definition. **Definition**

Let $X \subseteq U(\text{open}) \subseteq \mathbb{C}^n$ be a holomorphic subvariety.

Then X is **reducible** if $X = X_1 \cup X_2$ where X_1, X_2 are holomorphic subvarieties of U.
If X is nor reducible, then it is **irreducible**.

Current note has 0 direct children and 0 total descendants.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - Hol sets Holomorphic subsets of $\mathbb{C}^n$

And it has 36 siblings.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - 1Hol Holomorphic functions on spaces over $\mathbb{C}$ of dimension 1
    - circle packing Circle packing on $\mathbb{R}^2$
    - circle packing to Riemann map Circle packing converges to the Riemann biholomorphism
    - Cn conn open bounded Bounded connected open subsets of $\mathbb{C}^n$
    - Cn conn open circular Connected circular open subsets of $\mathbb{C}^n$
    - cont Continuous functions on $\mathbb{R}^d$
    - cube dyadic Dyadic cubes
    - curves Curves
    - derivative Differentiable functions
    - forms Differential forms on $\mathbb{R}^n$
    - Fourier-Wigner Fourier-Wigner transform
    - harmonic composed conformal Harmonic functions composed with conformal maps
    - Hilbert Hilbert transform
    - hol harmonic disk-circle Fourier-Cauchy-Poisson correspondence of holomorphic and harmonic functions on the unit disk and their boundary values
    - Hol sets Holomorphic subsets of $\mathbb{C}^n$
    - hypersurf 2n reg Regular hypersurfaces in $\mathbb{R}^{2n}$
    - hypersurf or Orientable hypersurfaces in $\mathbb{R}^n$
    - KG

$$\partial_t^2 + \sum_{i=1}^n v_i^2 \partial_{x_i}^2 + m^2$$

    - Laplace Laplace operator on $\mathbb{R}^n$
    - Lmeas Lebesgue measurable subsets of and functions on $\mathbb{R}^n, T^n, S^n$
    - Lmeas bd of open Lebesgue measure of boundary of open sets in $\mathbb{R}^n$
    - met density Metric density of subsets of $\mathbb{R}^n$
    - Mobius n-sphere Mobius endomorphisms

- monotone Monotone functions on $\mathbb{R}$
- periodic int Cauchy Cauchy integral of periodic functions
- poly int Polygons with integer vertices
- R 2 open smooth End Open smooth maps $U \subseteq \mathbb{R}^2 \to \mathbb{C}$
- R n discrete subg Discrete subgroups of $\mathbb{R}^n$
- R n discrete subg cocpt Discrete cocompact subgroups of $\mathbb{R}^n$, flat tori
- RC ramified germs Ramified germs of smooth and holomorphic functions
- Rn open Open subsets of $\mathbb{R}^n$
- Rn open Riem Open subsets of $\mathbb{R}^n$ equipped with the flat metric
- smooth quasi-analytic Quasi-analytic smooth functions on $\mathbb{R}$
- star shaped Star-shaped subsets of $\mathbb{R}^n$
- Vec ODEs in $\mathbb{R}^n \leftrightarrow$ Vector fields in $\mathbb{R}^n$
- wave

$$\partial_t^2 + \sum_{i=1}^{n} v_i^2 \partial_{x_i}^2$$