> **Info**
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Volterra operator $\int_{[0,-]} : L^p[a,b] \to L^p[a,b]$

Definition. **The Volterra operator is the $\mathbb{R}$-vector space endomorphism**

$$\begin{aligned} V : L^p[a,b] &\to L^p[a,b] \\ f &\mapsto (Vf)(x) := \int_{[0,x]} f \end{aligned}$$

The image is contained in $\mathcal{C}^0[a,b] \subseteq L^p[a,b]$?

the Volterra operator is bounded for $p = 1$

On $L^1[0,1]$ we have

$$\begin{aligned} |(Vf)(x)| &= \int_{[0,x]} f \\ (Vf)(x) &\leq \int_{[0,x]} |f| \\ &\leq \int_{[0,1]} |f| = \|f\|_1 \\ \|Vf\|_1^1 &\leq \|f\|_1 \end{aligned}$$

So $\|V\|_1 \leq 1$ now consider

$$\begin{aligned} \frac{\left\|V(n\chi_{[0,1/n]})\right\|_1}{\left\|n\chi_{[0,1/n]}\right\|_1} &= \frac{\int_{[0,1/n]} nx + \int_{[1/n,1]} 1}{\int_{[0,1/n]} n} \\ &= \frac{1}{2}\left(\frac{1}{n}\right)(1) + 1\left(1 - \frac{1}{n}\right) = 1 - \frac{1}{2n} \end{aligned}$$

Hence, as $n \to \infty$ this ratio goes to 1 so

$$\|V\|_1 = 1$$

the Volterra operator is bounded for $p = 2$

the Volterra operator is compact

the adjoint of the Volterra operator for $p = 2$

$$\langle Vf, g \rangle =$$

Current note has 0 direct children and 0 total descendants.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - Lmeas Lebesgue measurable subsets of and functions on $\mathbb{R}^n, T^n, S^n$
      - End int Volterra operator $\int_{[0,-]} : L^1_{\text{loc}}[0,1) \to \mathcal{C}[0,1)$
        - p to p Volterra operator $\int_{[0,-]} : L^p[a,b] \to L^p[a,b]$

And it has 1 siblings.

- stamp stamp
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