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$L^{p,q}$

Definition. $L^{p,q}$ spaces

Let

$$f : E \to \mathbb{C}$$

be a measurable function. Then the (p,q)-quasinorm for $(p,q) \in (0,\infty) \times (0,\infty]$ is

$$\|f\|_{p,q} := p^{\frac{1}{q}} \left\| t \mapsto t(m\{|f| \geq t\})^{\frac{1}{p}} \right\|_q$$

The **(p,q)-Lorentz space** is

$$L^{p,q}(E) := \left\{ f : E \to \mathbb{C} \middle| \|f\|_{p,q} < \infty \right\}$$

In particular,

$$L^{1,\infty}(\mathbb{R}) := \{f : \mathbb{R} \to \mathbb{C} \text{ measurable} \mid \sup_{\lambda > 0} \lambda m(f > \lambda) < \infty\}$$

Proposition:

(Chebyshev's inequality) Let $f \in L^p(\mathbb{R}^d), p \in (0,\infty)$. Then for any $\alpha > 0$

$$m\{|f| > \alpha\} \leq \frac{\|f\|_p^p}{\alpha^p}$$

$$
\begin{aligned}
\|f\|_p^p &= \int_{\mathbb{R}^d} |f|^p \\
&\geq \int_{\{|f|>\alpha\}} |f|^p \\
&\geq \alpha^p m\{|f| > \alpha\}
\end{aligned}
$$

implies

$$L^1(\mathbb{R}) \subseteq L^{1,\infty}(\mathbb{R})$$

Current note has 0 direct children and 0 total descendants.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - Lmeas Lebesgue measurable subsets of and functions on $\mathbb{R}^n, T^n, S^n$
      - Lorentz $L^{p,q}$

And it has 15 siblings.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - Lmeas Lebesgue measurable subsets of and functions on $\mathbb{R}^n, T^n, S^n$
      - BMO Functions with uniformly bounded mean oscillations on cubes
      - decom bd Chebyshev's inequality
      - decom CZ Calderon-Zygmund decomposition
      - density Lebesgue density of measurable sets
      - DMO Functions with uniformly bounded mean oscillations on dyadic cubes
      - End int Volterra operator $\int_{[0,-]} : L^1_{\text{loc}}[0,1) \to \mathcal{C}[0,1)$
      - f Measurable functions on $\mathbb{R}^n$
      - f quant (ϵ, n)-measurable function
      - int Integrability and integral of measurable functions on $\mathbb{R}^n$
      - int HL Hardy-Littlewood maximal functions of $L^1_{\text{loc}}(\mathbb{R}^n)$
      - int mean Lebesgue averaging and differentiation
      - int monotone Integrals of a monotonically converging sequence of functions
      - int undergraph Lebesgue integral from measure of undergraph
      - Lorentz $L^{p,q}$

- unit mass $\mathrm{E}([0,1]) = \mathrm{E}(0,1), \mathrm{E}([-\pi,\pi]) = \mathrm{E}(S^1)$