Info

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Lebesgue density of measurable sets

Definition. Density of measurable sets

Let $E \subseteq \mathbb{R}^d$ be a measurable set. Then

$$\text{density}_E(x) := \lim_{\epsilon \to 0} \frac{m(B_\epsilon(x) \cap E)}{m(B_\epsilon(x))}$$

is the **density** of E at $x \in \mathbb{R}^d$. The **density** set of E is the set of all points where the density of E is 1

$$\text{dens}(E) := \left\{ x \in E \middle| \lim_{x \in B; m(B) \to 0} \frac{m(B \cap E)}{m(B)} = 1 \right\}$$

(Lebesgue density theorem) For almost every $x \in E$, we have

$$\text{density}_E(x) = 1$$

using differentiation

(Differentiation of the Lebesgue integral on balls) If $f \in L^1_{\text{loc}}(\mathbb{R}^d)$ then

$$\lim_{r \to 0} (A_r f)(x) = f(x) \text{ aeon } x \in \mathbb{R}^d$$

Current note has 0 direct children and 0 total descendants.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - Lmeas Lebesgue measurable subsets of and functions on $\mathbb{R}^n, T^n, S^n$
      - density Lebesgue density of measurable sets

And it has 15 siblings.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - Lmeas Lebesgue measurable subsets of and functions on $\mathbb{R}^n, T^n, S^n$
      - BMO Functions with uniformly bounded mean oscillations on cubes
      - decom bd Chebyshev's inequality
      - decom CZ Calderon-Zygmund decomposition
      - density Lebesgue density of measurable sets
      - DMO Functions with uniformly bounded mean oscillations on dyadic cubes
      - End int Volterra operator $\int_{[0,-]} : L^1_{\mathrm{loc}}[0,1) \to \mathcal{C}[0,1)$
      - f Measurable functions on $\mathbb{R}^n$
      - f quant (ϵ, n)-measurable function
      - int Integrability and integral of measurable functions on $\mathbb{R}^n$
      - int HL Hardy-Littlewood maximal functions of $L^1_{\mathsf{loc}}(\mathbb{R}^n)$
      - int mean Lebesgue averaging and differentiation
      - int monotone Integrals of a monotonically converging sequence of functions
      - int undergraph Lebesgue integral from measure of undergraph
      - Lorentz $L^{p,q}$
      - unit mass $\mathrm{E}([0,1]) = \mathrm{E}(0,1), \mathrm{E}([-\pi,\pi]) = \mathrm{E}(S^1)$