Info

This note found here
as a part of a collection
is written (completely with human hands) by Rupadarshi Ray,
created on February 8, 2024 9:09:29 AM,
and was last modified on June 10, 2026 8:16:11 PM.

Measurable functions on $\mathbb{R}^n$

Definition. Measurable functions on $\mathbb{R}^n$

Let $E \subseteq \mathbb{R}^n$ be a measurable subset. A function $f : E \to \mathbb{R}$ is **measurable** if

$$f^{-1}(-\infty, a) = \{f < a\}$$

is measurable for all $a \in \mathbb{R}$.

Let

$$f : \mathbb{R}^d \to [0, \infty)$$

be a measurable function. Then there exists a sequence of simple functions $\{\varphi_n : \mathbb{R}^d \to [0, \infty)\}$ such that

$$\phi_n \underset{n\to\infty}{\nearrow} f$$

Consider the N-th truncation of f

$$F_N := f1_{[0,N]^d \cap \{f \leq N\}} + N1_{[0,N]^d \cap \{f > N\}}$$

which is bounded

$$F_N : \mathbb{R}^d \to [0, N]$$

has support $\subseteq [0, N]^d$ and converges to f pointwise

$$F_N \xrightarrow{N\to\infty} f$$

- Now we parition the codomain $[0, N]$ of F_N into MN parts and approximate F_N by the simple function

$$F_{N,M} := \sum_{0\le l\le NM} \frac{l}{M} 1_{\{\frac{l}{M}<F_N\le\frac{l+1}{M}\}}$$

- This implies

$$0 \le F_N - F_{N,M} \le \frac{1}{M}$$

- Then

$$\phi_k := F_{2^k,2^k} \underset{n\to\infty}{\nearrow} f$$

(Ergorov) Let $(f_{n)}$ be a sequence of measurable function on $E \subseteq \mathbb{R}^d$ with $m(E) < \infty$ and

$$f_n \to f \text{ aeon } E$$

Then for any given $\epsilon > 0$ we can find a closed set $A_\epsilon \subset E$ such that $m(E - A_\epsilon) \le \epsilon$ and

$$f_n \xrightarrow{\text{uniformly}} f \text{ on } E$$

(Lusin) Let f be measurable function on $E \subseteq \mathbb{R}^d$ with $m(E) < \infty$. Then for every $\epsilon > 0$ there exists a closed set $F_\epsilon \subset E$ such that $m(E - F_\epsilon) \le \epsilon$ so that

$$f\Big|_{F_\epsilon}$$

is continuous.

Current note has 0 direct children and 0 total descendants.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - Lmeas Lebesgue measurable subsets of and functions on $\mathbb{R}^n, T^n, S^n$
      - f Measurable functions on $\mathbb{R}^n$

And it has 15 siblings.

- stamp stamp

- Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
  - Lmeas Lebesgue measurable subsets of and functions on $\mathbb{R}^n, T^n, S^n$
    - BMO Functions with uniformly bounded mean oscillations on cubes
    - decom bd Chebyshev's inequality
    - decom CZ Calderon-Zygmund decomposition
    - density Lebesgue density of measurable sets
    - DMO Functions with uniformly bounded mean oscillations on dyadic cubes
    - End int Volterra operator $\int_{[0,-]} : L^1_{\mathrm{loc}}[0,1) \to \mathcal{C}[0,1)$
    - f Measurable functions on $\mathbb{R}^n$
    - f quant (ϵ, n)-measurable function
    - int Integrability and integral of measurable functions on $\mathbb{R}^n$
    - int HL Hardy-Littlewood maximal functions of $L^1_{\text{loc}}(\mathbb{R}^n)$
    - int mean Lebesgue averaging and differentiation
    - int monotone Integrals of a monotonically converging sequence of functions
    - int undergraph Lebesgue integral from measure of undergraph
    - Lorentz $L^{p,q}$
    - unit mass $\mathrm{E}([0,1]) = \mathrm{E}(0,1), \mathrm{E}([-\pi,\pi]) = \mathrm{E}(S^1)$