

### Info

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## Lebesgue integral from measure of undergraph

### Definition.

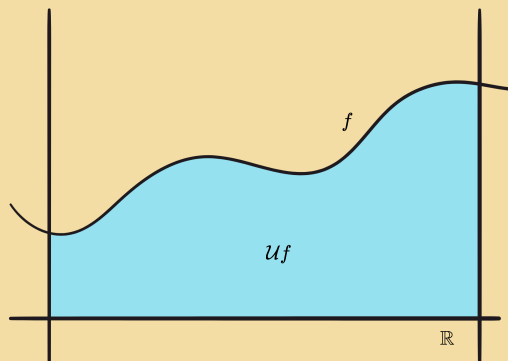
$$\mathcal{U}f := \{(x, y) \in \mathbb{R}^d \times \mathbb{R} \mid 0 \leq y < f(x)\}$$

### Let

$$f : \mathbb{R}^d \rightarrow [0, \infty)$$

be a measurable positive function. Then the Lebesgue integral is the measure of the undergraph in  $\mathbb{R}^{d+1}$

$$\int_{\mathbb{R}^d} f = m_{\mathbb{R}^{d+1}}(\mathcal{U}f)$$



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      - [DMO](#) Functions with uniformly bounded mean oscillations on dyadic cubes
      - [End int](#) Volterra operator  $\int_{[0,-]} : L^1_{\text{loc}}[0, 1] \rightarrow \mathcal{C}[0, 1]$
      - [f](#) Measurable functions on  $\mathbb{R}^n$
      - [f quant](#)  $(\epsilon, n)$ -measurable function
      - [int](#) Integrability and integral of measurable functions on  $\mathbb{R}^n$
      - [int HL](#) Hardy-Littlewood maximal functions of  $L^1_{\text{loc}}(\mathbb{R}^n)$
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      - [int monotone](#) Integrals of a monotonically converging sequence of functions
      - [int undergraph](#) Lebesgue integral from measure of undergraph
      - [Lorentz](#)  $L^{p,q}$
      - [unit mass](#)  $\mathbf{E}([0, 1]) = \mathbf{E}(0, 1), \mathbf{E}([- \pi, \pi]) = \mathbf{E}(S^1)$