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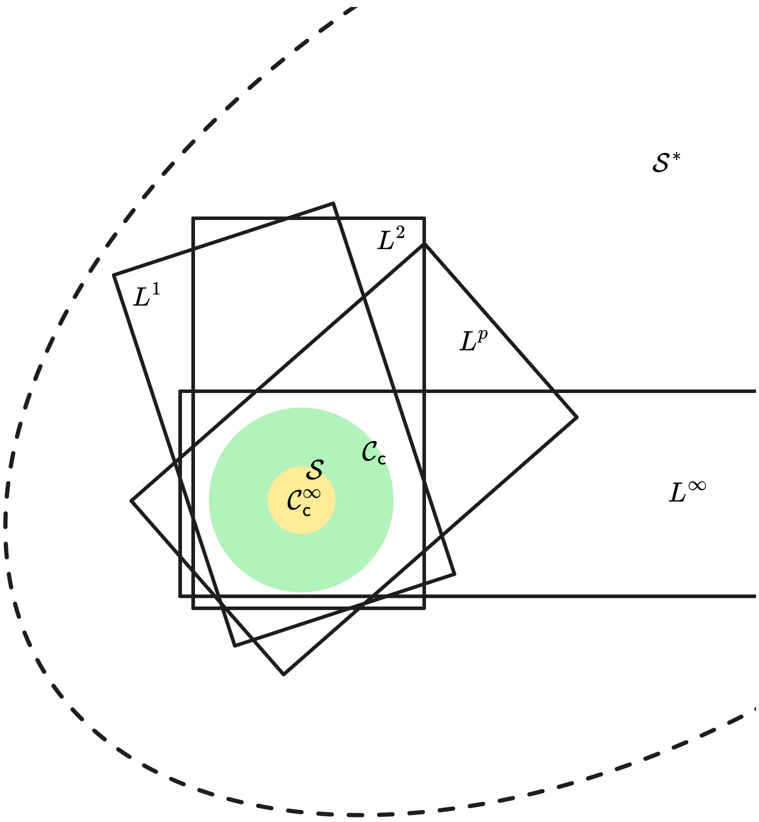
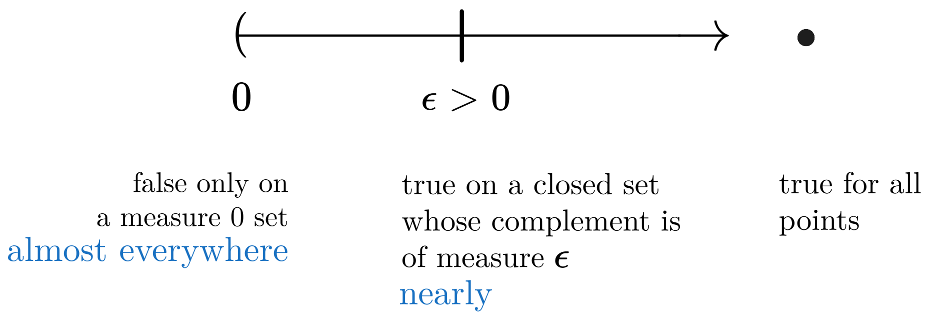
Lebesgue measurable subsets of and functions on $\mathbb{R}^n, T^n, S^n$



New notions on equality and properties!

- Some property **p** is true for elements of a measurable set E *almost everywhere* if it is only false on a measure 0 set.
- Some property **p** is *nearly true* for elements of a measurable set E if for all $\epsilon > 0$ there exists a closed set $F_\epsilon \subseteq E$ such that $m(E \setminus F_\epsilon) \leq \epsilon$ and it is true on F_ϵ.



Littlewood's three principles about measurable sets and functions say

- Every set is *nearly(?)* a finite union of intervals.
- Every convergent sequence of measurable functions is *nearly* uniformly convergent.

  (Ergorov) Let (f_{n}) be a sequence of measurable function on $E \subseteq \mathbb{R}^d$ with $m(E) < \infty$ and

  $$f_n \to f \text{ aeon } E$$

  Then for any given $\epsilon > 0$ we can find a closed set $A_\epsilon \subset E$ such that $m(E - A_\epsilon) \leq \epsilon$ and

  $$f_n \xrightarrow{\text{uniformly}} f \text{ on } E$$

- Every measurable function is *nearly* continuous.

  (Lusin) Let f be measurable function on $E \subseteq \mathbb{R}^d$ with $m(E) < \infty$. Then for every $\epsilon > 0$ there exists a closed set $F_\epsilon \subset E$ such that $m(E - F_\epsilon) \leq \epsilon$ so that

  $$f\Big|_{F_\epsilon}$$

  is continuous.

subsets	functions
	Definition. Measurable functions on $\mathbb{R}^n$ Let $E \subseteq \mathbb{R}^n$ be a measurable subset. A function $f : E \to \mathbb{R}$ is **measurable** if

subsets	functions
	$$f^{-1}(-\infty, a) = \{f < a\}$$ is measurable for all $a \in \mathbb{R}$.
(Lebesgue density theorem) For almost every $x \in E$, we have $$\text{density}_E(x) = 1$$	**(Differentiation of the Lebesgue integral on balls) If $f \in L^1_{\text{loc}}(\mathbb{R}^d)$ then** $$\lim_{r \to 0}(A_r f)(x) = f(x) \text{ aeon } x \in \mathbb{R}^d$$

Current note has 15 direct children and 16 total descendants.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - Lmeas Lebesgue measurable subsets of and functions on $\mathbb{R}^n, T^n, S^n$
      - BMO Functions with uniformly bounded mean oscillations on cubes
      - decom bd Chebyshev's inequality
      - decom CZ Calderon-Zygmund decomposition
      - density Lebesgue density of measurable sets
      - DMO Functions with uniformly bounded mean oscillations on dyadic cubes
      - End int Volterra operator $\int_{[0,-]} : L^1_{\text{loc}}[0, 1) \to \mathcal{C}[0, 1)$
        - p to p Volterra operator $\int_{[0,-]} : L^p[a, b] \to L^p[a, b]$
      - f Measurable functions on $\mathbb{R}^n$
      - f quant (ϵ, n)-measurable function
      - int Integrability and integral of measurable functions on $\mathbb{R}^n$
      - int HL Hardy-Littlewood maximal functions of $L^1_{\text{loc}}(\mathbb{R}^n)$
      - int mean Lebesgue averaging and differentiation
      - int monotone Integrals of a monotonically converging sequence of functions
      - int undergraph Lebesgue integral from measure of undergraph
      - Lorentz $L^{p,q}$
      - unit mass $\mathrm{E}([0, 1]) = \mathrm{E}(0, 1), \mathrm{E}([-\pi, \pi]) = \mathrm{E}(S^1)$

And it has 36 siblings.

- stamp stamp

- Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
  - 1Hol Holomorphic functions on spaces over $\mathbb{C}$ of dimension 1
  - circle packing Circle packing on $\mathbb{R}^2$
  - circle packing to Riemann map Circle packing converges to the Riemann biholomorphism
  - Cn conn open bounded Bounded connected open subsets of $\mathbb{C}^n$
  - Cn conn open circular Connected circular open subsets of $\mathbb{C}^n$
  - cont Continuous functions on $\mathbb{R}^d$
  - cube dyadic Dyadic cubes
  - curves Curves
  - derivative Differentiable functions
  - forms Differential forms on $\mathbb{R}^n$
  - Fourier-Wigner Fourier-Wigner transform
  - harmonic composed conformal Harmonic functions composed with conformal maps
  - Hilbert Hilbert transform
  - hol harmonic disk-circle Fourier-Cauchy-Poisson correspondence of holomorphic and harmonic functions on the unit disk and their boundary values
  - Hol sets Holomorphic subsets of $\mathbb{C}^n$
  - hypersurf 2n reg Regular hypersurfaces in $\mathbb{R}^{2n}$
  - hypersurf or Orientable hypersurfaces in $\mathbb{R}^n$
  - KG

$$\partial_t^2 + \sum_{i=1}^{n} v_i^2 \partial_{x_i}^2 + m^2$$

  - Laplace Laplace operator on $\mathbb{R}^n$
  - Lmeas Lebesgue measurable subsets of and functions on $\mathbb{R}^n, T^n, S^n$
  - Lmeas bd of open Lebesgue measure of boundary of open sets in $\mathbb{R}^n$
  - met density Metric density of subsets of $\mathbb{R}^n$
  - Mobius n-sphere Mobius endomorphisms
  - monotone Monotone functions on $\mathbb{R}$
  - periodic int Cauchy Cauchy integral of periodic functions
  - poly int Polygons with integer vertices
  - R 2 open smooth End Open smooth maps $U \subseteq \mathbb{R}^2 \to \mathbb{C}$
  - R n discrete subg Discrete subgroups of $\mathbb{R}^n$
  - R n discrete subg cocpt Discrete cocompact subgroups of $\mathbb{R}^n$, flat tori
  - RC ramified germs Ramified germs of smooth and holomorphic functions
  - Rn open Open subsets of $\mathbb{R}^n$
  - Rn open Riem Open subsets of $\mathbb{R}^n$ equipped with the flat metric

- smooth quasi-analytic Quasi-analytic smooth functions on $\mathbb{R}$
- star shaped Star-shaped subsets of $\mathbb{R}^n$
- Vec ODEs in $\mathbb{R}^n$ $\leftrightarrow$ Vector fields in $\mathbb{R}^n$
- wave

$$\partial_t^2 + \sum_{i=1}^{n} v_i^2 \partial_{x_i}^2$$