Info

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Lebesgue measure of boundary of open sets in $\mathbb{R}^n$

for any $\alpha, \beta > 0$ there exists an open set of measure $\leq \alpha$ whose boundary has measure $\geq \beta$

- Let $\alpha, \beta > 0$ and K be closure of an open set such that $m(K) = \alpha + \beta$.
  - Let

    $$\{s_m\}_{m \in \mathbb{N}} \subseteq K$$

    be a countable dense sequence.
  - Consider a sequence $\{\alpha(m)\}_{m \in \mathbb{N}} \subseteq (0, \infty)$ such that

    $$m(B_1) \sum_{n \in \mathbb{N}} \alpha(n)^n = \alpha$$

    for example

    $$\alpha(n)^n :\equiv \frac{\alpha}{m(B_1)} \frac{1}{2^{n-1}}$$

  - Then we have

    $$U := \bigcup_{m \in \mathbb{N}} B_{\alpha(m)}(s_m) \cap K \subseteq K$$

  - So U is an open set with $\overline{U} = K$ and

    $$m(U) \leq m(B_1) \sum_{n \in \mathbb{N}} \alpha(m)^n = \alpha$$

    and

    $$\begin{aligned} \operatorname{int}(U) \sqcup \partial U &= \overline{U} \\ U \sqcup \partial U &= K \\ \underbrace{m(U)}_{\leq \alpha} + m(\partial U) &= \underbrace{m(K)}_{\alpha + \beta} \end{aligned}$$

- Therefore, for arbitrary $\alpha, \beta > 0$ we have an open set U of measure α whose boundary ∂U has measure $\geq \beta$.

[1]

Current note has 0 direct children and 0 total descendants.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - Lmeas bd of open Lebesgue measure of boundary of open sets in $\mathbb{R}^n$

And it has 36 siblings.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - 1Hol Holomorphic functions on spaces over $\mathbb{C}$ of dimension 1
    - circle packing Circle packing on $\mathbb{R}^2$
    - circle packing to Riemann map Circle packing converges to the Riemann biholomorphism
    - Cn conn open bounded Bounded connected open subsets of $\mathbb{C}^n$
    - Cn conn open circular Connected circular open subsets of $\mathbb{C}^n$
    - cont Continuous functions on $\mathbb{R}^d$
    - cube dyadic Dyadic cubes
    - curves Curves
    - derivative Differentiable functions
    - forms Differential forms on $\mathbb{R}^n$
    - Fourier-Wigner Fourier-Wigner transform
    - harmonic composed conformal Harmonic functions composed with conformal maps
    - Hilbert Hilbert transform
    - hol harmonic disk-circle Fourier-Cauchy-Poisson correspondence of holomorphic and harmonic functions on the unit disk and their boundary values
    - Hol sets Holomorphic subsets of $\mathbb{C}^n$
    - hypersurf 2n reg Regular hypersurfaces in $\mathbb{R}^{2n}$
    - hypersurf or Orientable hypersurfaces in $\mathbb{R}^n$
    - KG

$$\partial_t^2 + \sum_{i=1}^{n} v_i^2 \partial_{x_i}^2 + m^2$$

    - Laplace Laplace operator on $\mathbb{R}^n$
    - Lmeas Lebesgue measurable subsets of and functions on $\mathbb{R}^n, T^n, S^n$

- Lmeas bd of open Lebesgue measure of boundary of open sets in $\mathbb{R}^n$
- met density Metric density of subsets of $\mathbb{R}^n$
- Mobius n-sphere Mobius endomorphisms
- monotone Monotone functions on $\mathbb{R}$
- periodic int Cauchy Cauchy integral of periodic functions
- poly int Polygons with integer vertices
- R 2 open smooth End Open smooth maps $U \subseteq \mathbb{R}^2 \to \mathbb{C}$
- R n discrete subg Discrete subgroups of $\mathbb{R}^n$
- R n discrete subg cocpt Discrete cocompact subgroups of $\mathbb{R}^n$, flat tori
- RC ramified germs Ramified germs of smooth and holomorphic functions
- Rn open Open subsets of $\mathbb{R}^n$
- Rn open Riem Open subsets of $\mathbb{R}^n$ equipped with the flat metric
- smooth quasi-analytic Quasi-analytic smooth functions on $\mathbb{R}$
- star shaped Star-shaped subsets of $\mathbb{R}^n$
- Vec ODEs in $\mathbb{R}^n$ $\leftrightarrow$ Vector fields in $\mathbb{R}^n$
- wave

$$\partial_t^2 + \sum_{i=1}^{n} v_i^2 \partial_{x_i}^2$$

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1. analysis - Does the boundary of an open set have measure zero (in $\mathbb{R}^n$)? - Mathematics Stack Exchange ↩