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Mobius endomorphisms on $\overline{\mathbb{R}^n}^{\infty} \cong S^n$

Definition. Metric on $\overline{\mathbb{R}^n}^{\infty}$

$$d(x,y) := \begin{cases} \dfrac{2\,|x-y|}{\sqrt{1+|x|^2}\sqrt{1+|y|^2}} & x,y \neq \infty \\ \dfrac{2}{\sqrt{1+|x|^2}} & y = \infty \end{cases}$$

which gives the cross-ratio

$$[x,y,u,v] = \frac{|x-u|\,|y-u|}{|x-y|\,|u-v|}$$

that agrees with the cross ratio for the standard metric on $\mathbb{R}^n$

Definition. Mobius endomorphisms

- Let $rS^{n-1} + a$ be the sphere of radius $r > 0$ centered at $a \in \mathbb{R}^n$. Then the **inversion through the sphere** is the map

$$\begin{aligned} \mathbb{R}^n \cup \{\infty\} &\to \mathbb{R}^n \cup \{\infty\} \\ x &\mapsto a + \left(\frac{r}{|x-a|}\right)^2 (x-a) \\ &= a + r^2(x-a)^* \end{aligned}$$

- Let $t + \mathbb{R}\langle a\rangle^{\perp} \cup \{\infty\}$ be the extended plane orthogonal to $a \in \mathbb{R}^n \setminus \{0\}$ at a distance $t \in \mathbb{R}$. The **inversion through the plane** is the map

$$\begin{aligned} \mathbb{R}^n \cup \{\infty\} &\to \mathbb{R}^n \cup \{\infty\} \\ x &\mapsto x - 2(x \cdot a - t)a^* \end{aligned}$$

$$O^+(1, n+1) \cong \text{Moeb}(S^n) \curvearrowright S^n$$

dimension counts

$\dim S^n = n$	$\dim O^+(1, n+1) = \frac{(n}{}$	$\alpha(n) \cdot n$	$\text{Moeb}(S^n)_\circ$
2	6	$3 \cdot 2 = 6$	sharply 3-transitive
3	10	$3 \cdot 3 = 9$	
4	15	$3 \cdot 4 = 12$	
5	21	$4 \cdot 5 = 20$	
...			

Current note has 0 direct children and 0 total descendants.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - Mobius n-sphere Mobius endomorphisms

And it has 36 siblings.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - 1Hol Holomorphic functions on spaces over $\mathbb{C}$ of dimension 1
    - circle packing Circle packing on $\mathbb{R}^2$
    - circle packing to Riemann map Circle packing converges to the Riemann biholomorphism
    - Cn conn open bounded Bounded connected open subsets of $\mathbb{C}^n$
    - Cn conn open circular Connected circular open subsets of $\mathbb{C}^n$
    - cont Continuous functions on $\mathbb{R}^d$
    - cube dyadic Dyadic cubes
    - curves Curves
    - derivative Differentiable functions
    - forms Differential forms on $\mathbb{R}^n$
    - Fourier-Wigner Fourier-Wigner transform

- harmonic composed conformal Harmonic functions composed with conformal maps
- Hilbert Hilbert transform
- hol harmonic disk-circle Fourier-Cauchy-Poisson correspondence of holomorphic and harmonic functions on the unit disk and their boundary values
- Hol sets Holomorphic subsets of $\mathbb{C}^n$
- hypersurf 2n reg Regular hypersurfaces in $\mathbb{R}^{2n}$
- hypersurf or Orientable hypersurfaces in $\mathbb{R}^n$
- KG

$$\partial_t^2 + \sum_{i=1}^{n} v_i^2 \partial_{x_i}^2 + m^2$$

- Laplace Laplace operator on $\mathbb{R}^n$
- Lmeas Lebesgue measurable subsets of and functions on $\mathbb{R}^n, T^n, S^n$
- Lmeas bd of open Lebesgue measure of boundary of open sets in $\mathbb{R}^n$
- met density Metric density of subsets of $\mathbb{R}^n$
- Mobius n-sphere Mobius endomorphisms
- monotone Monotone functions on $\mathbb{R}$
- periodic int Cauchy Cauchy integral of periodic functions
- poly int Polygons with integer vertices
- R 2 open smooth End Open smooth maps $U \subseteq \mathbb{R}^2 \to \mathbb{C}$
- R n discrete subg Discrete subgroups of $\mathbb{R}^n$
- R n discrete subg cocpt Discrete cocompact subgroups of $\mathbb{R}^n$, flat tori
- RC ramified germs Ramified germs of smooth and holomorphic functions
- Rn open Open subsets of $\mathbb{R}^n$
- Rn open Riem Open subsets of $\mathbb{R}^n$ equipped with the flat metric
- smooth quasi-analytic Quasi-analytic smooth functions on $\mathbb{R}$
- star shaped Star-shaped subsets of $\mathbb{R}^n$
- Vec ODEs in $\mathbb{R}^n$ $\leftrightarrow$ Vector fields in $\mathbb{R}^n$
- wave

$$\partial_t^2 + \sum_{i=1}^{n} v_i^2 \partial_{x_i}^2$$