

Info

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is written (completely with human hands) by [Rupadarshi Ray](#),
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Total variation and sequence of continuous functions



$$\lim_{n \rightarrow \infty} f_n(x) = f(x) \implies V_{[a,b]}(f) \leq \liminf_{n \rightarrow \infty} V_{[a,b]}(f_n)$$

Fix a partition $(x_k)_{k=1}^N$ of $[a, b]$ and let $\epsilon > 0$. Then there is a M such that

$$\begin{aligned} M < n &\implies |f_n(x_k) - f(x_k)| < \epsilon/2 \\ &|f_n(x_{k-1}) - f(x_{k-1})| < \epsilon/2 \end{aligned}$$

$$\begin{aligned} f(x_k) - f(x_{k-1}) &= f(x_k) - f_n(x_k) \\ &\quad + f_n(x_k) - f_n(x_{k-1}) \\ &\quad + f_n(x_{k-1}) - f(x_{k-1}) \\ |f(x_k) - f(x_{k-1})| &= |f(x_k) - f_n(x_k)| \\ &\quad + |f_n(x_k) - f_n(x_{k-1})| \\ &\quad + |f_n(x_{k-1}) - f(x_{k-1})| \\ &\leq \epsilon/2 + |f_n(x_k) - f_n(x_{k-1})| + \epsilon/2 \\ \sum_{k=1}^N |f(x_k) - f(x_{k-1})| &\leq N\epsilon + \sum_{k=1}^N |f_n(x_k) - f_n(x_{k-1})| \\ &\leq \lim_{n \rightarrow \infty} \sum_{k=1}^N |f_n(x_k) - f_n(x_{k-1})| \end{aligned}$$

Because

$$\begin{aligned} \sum_{k=1}^N |f_n(x_k) - f_n(x_{k-1})| &\leq V_{[a,b]}(f_n), \forall n > 0 \\ \liminf_{n \rightarrow \infty} \sum_{k=1}^N |f_n(x_k) - f_n(x_{k-1})| &\leq \liminf_{n \rightarrow \infty} V_{[a,b]}(f_n) \end{aligned}$$

So we have

$$\begin{aligned}\sum_{k=1}^N |f(x_k) - f(x_{k-1})| &\leq \lim_{n \rightarrow \infty} \sum_{k=1}^N |f_n(x_k) - f_n(x_{k-1})| \\ &= \liminf_{n \rightarrow \infty} \sum_{k=1}^N |f_n(x_k) - f_n(x_{k-1})| \\ &\leq \liminf_{n \rightarrow \infty} V_{[a,b]}(f_n)\end{aligned}$$

Thus

$$V_{[a,b]}(f) := \sup_{\text{partitions}} \sum_{k=1}^N |f(x_k) - f(x_{k-1})| \leq \liminf_{n \rightarrow \infty} V_{[a,b]}(f_n)$$

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