Info

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Extremum of functions $\mathbb{R} \to \mathbb{R}$

Definition. Local and global extremum of a function from a metric space to $\mathbb{R}$

Given a function $f : X \to \mathbb{R}$, then at $x \in X$ f has a

- **global maxima** if $f(x) \geq f(a)$ for all $a \in X$
- **global minima** if $f(x) \leq f(a)$ for all $a \in X$
- **local maxima** if $f(x) \geq f(a)$ for all $a \in \epsilon\text{–}B(x)$ for some $\epsilon > 0$
- **local minima** if $f(x) \leq f(a)$ for all $a \in \epsilon\text{–}B(x)$ for some $\epsilon > 0$

differentiable functions on open interval

(Derivative, if it exists, must be 0 at local extremas) Let a function $f : (a, b) \to \mathbb{R}$ have a local maxima or minima at $c \in (a, b)$. If f has an extended derivative at c then the derivative must be zero

$$f'(c) \in \mathbb{R} \cup \{-\infty, \infty\} \implies f'(c) = 0$$

Therefore,

$$\{\text{local extremum of } f\} \subseteq Z(f')$$

The reverse of this implication is not true, because f may have an extremum without being differentiable.

Even if $f'(c) = 0$, $f(c)$ might not be a local maxima or a minima for f because of the following simple example.

$$x^3 : \mathbb{R} \to \mathbb{R}$$

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x^3
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Here, the derivative is $3x^2$ which is 0 at 0 but the function is increasing and does not have any local extremum.

continuous functions on closed internal

(Extremum value theorem for continuous $[a, b] \to \mathbb{R}$) Let

$$f : [a, b] \to \mathbb{R}$$

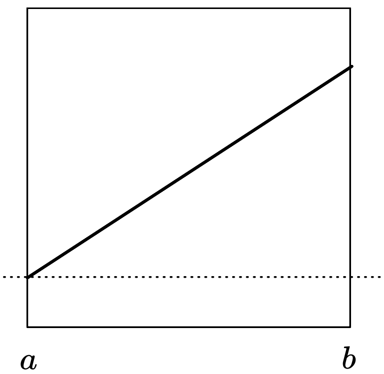
be continuous, then a global maxima *and* a minima exists $M, m \in [a, b]$ for f.

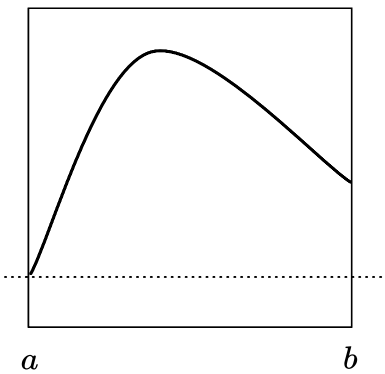
By

A continuous function from a compact metric space to $\mathbb{R}$ has a global maxima *and* a global minima.

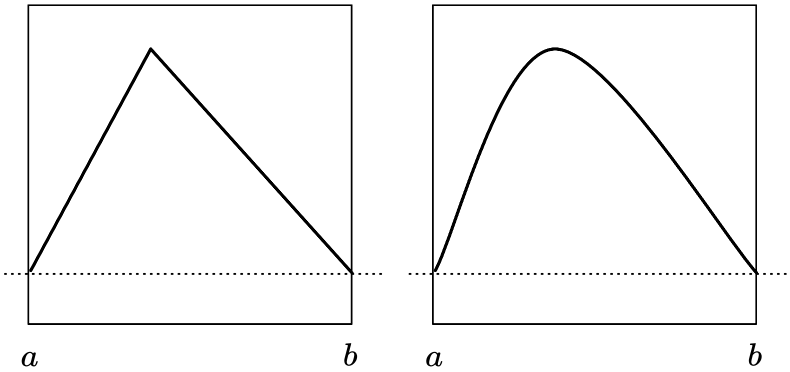
- Image is a compact subset of $\mathbb{R}$, so it closed and bounded, so its sup and inf is in the set.
- The sup and inf are global maxima and minima.

- The extremums might be on the endpoints a, b, or in (a, b).
- If $f(M) = f(m)$ then the function is constant.
- If $M = m$ then $f(M) = f(m)$, hence the function is constant.





differentiable functions on closed intervals with fixed endpoints



derivative tests

Current note has 0 direct children and 0 total descendants.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - R1
      - extremum Extremum of functions $\mathbb{R} \to \mathbb{R}$

And it has 5 siblings.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - R1
      - discont types Types of discontinuities of functions on $\mathbb{R}$
      - extremum Extremum of functions $\mathbb{R} \to \mathbb{R}$
      - fixed pts Fixed points of functions $U \subseteq \mathbb{R} \to U$
      - limit of fractions $\lim_{x\to a} \frac{f(x)}{g(x)}$ of functions $f, g : U \subseteq \mathbb{R} \to \mathbb{R}$
      - Var and sequence of functions Total variation and sequence of continuous functions