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Ramified germs of smooth and holomorphic functions

Definition. Ramified germs of holomorphic functions

With the stalk of holomorphic functions at $0 \in \mathbb{C}^n$, $\mathscr{O}_0(\mathbb{C}^n) \equiv \mathscr{O}_n$ and the ideal $\mathfrak{m}_n \subseteq \mathscr{O}_n$ of such functions vanishing at 0, let the group of germs(?) of biholomorphisms $\mathscr{D}_n$ act on $\mathscr{O}_n$ by $f \overset{g}{\mapsto} f \circ g^{-1}$. Then

- a **critical point** at 0 is a germ $f \in \mathscr{O}_n$ such that $f'(0) = 0$
- a **singularity** is a equivalence class $[f] \in \mathscr{O}_n / \mathscr{D}_n$

Definition. Stable equivalence of germs

The germs $f \in \mathfrak{m}_{n_1}, g \in \mathfrak{m}_{n_2}$ are **stably equivalent** if

$$f + x_{n_1+1}^2 + \cdots + x_k^2$$

is equivalent to g

$$\frac{\coprod_{n \geq 1} \mathfrak{m}_n}{\text{stable equivalence}}$$

Definition. Versal deformations of a critical point

A **deformation with base $\mathbb{C}^l$ of the germ $f \in \mathscr{O}_n$** is the germ at $0 \in \mathbb{C}^n \times \mathbb{C}^l$ of a smooth map

$$F : (\mathbb{C}^n \times \mathbb{C}^l, 0) \to \mathbb{C},\ F(-, 0) = f$$

- two deformations F_1, F_2 are **equivalent** if

$$F_1(x, \lambda) = F_2(g_\lambda(x), \lambda)$$

where $g_\lambda : (\mathbb{C}^n \times \mathbb{C}^l, 0) \to (\mathbb{C}^n, 0)$ is a smooth germ of $\lambda \in \mathbb{C}^n$ parameter family of diffeomorphisms

- the deformation F_2 is **induced** from F_1 is

$$F_2(x, \lambda) = F_1(x, \theta(\lambda))$$

where $\theta : (\mathbb{C}^{l_1}, 0) \to (\mathbb{C}^{l_2}, 0)$ is a smooth germ of a mapping of the bases

- deformation F of a germ f is said to be **versal** if every deformation F' of f can be represented of the form

$$F'(x, \lambda) = F(g(x, \lambda), \theta(\lambda)),\ g(x, 0) = x,\ \theta(0) = 0$$

that is, every deformation of f is *equivalent* to a deformation *induced* from f

Deformations are "perturbations".

Definition. **Modality of a critical point**

The group of germs of diffeomorphisms act on the function space $\mathfrak{m}_n \subset \mathscr{O}_n$ of functions $f : (\mathbb{C}^n, 0) \to (\mathbb{C}, 0)$. The modality $\mathrm{md}(f)$ of such a function germ is the modality of any of its jets $j^k f, k \geq \mu(f) + 1$.
Modality is the minimum k such that a sufficiently small nbd of f is covered by a finite number of k-parameter families of orbits.

Does this number answer how many *inequivalent "perturbations"* of f are there?

Definition. **Level bifurcation set of a singularity**

classification of singularities of holomorphic functions upto stable equivalence

- stamp.Rf.RC ramified germs.Hol modality 1

modality $= 1$			
modality $= 2$			

Current note has 1 direct children and 1 total descendants.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - RC ramified germs Ramified germs of smooth and holomorphic functions
      - Hol modality 1 Simple singularities of holomorphic functions

And it has 36 siblings.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - 1Hol Holomorphic functions on spaces over $\mathbb{C}$ of dimension 1
    - circle packing Circle packing on $\mathbb{R}^2$
    - circle packing to Riemann map Circle packing converges to the Riemann biholomorphism
    - Cn conn open bounded Bounded connected open subsets of $\mathbb{C}^n$
    - Cn conn open circular Connected circular open subsets of $\mathbb{C}^n$
    - cont Continuous functions on $\mathbb{R}^d$
    - cube dyadic Dyadic cubes
    - curves Curves
    - derivative Differentiable functions
    - forms Differential forms on $\mathbb{R}^n$
    - Fourier-Wigner Fourier-Wigner transform
    - harmonic composed conformal Harmonic functions composed with conformal maps
    - Hilbert Hilbert transform
    - hol harmonic disk-circle Fourier-Cauchy-Poisson correspondence of holomorphic and harmonic functions on the unit disk and their boundary values
    - Hol sets Holomorphic subsets of $\mathbb{C}^n$
    - hypersurf 2n reg Regular hypersurfaces in $\mathbb{R}^{2n}$
    - hypersurf or Orientable hypersurfaces in $\mathbb{R}^n$
    - KG

      $$\partial_t^2 + \sum_{i=1}^{n} v_i^2 \partial_{x_i}^2 + m^2$$

    - Laplace Laplace operator on $\mathbb{R}^n$
    - Lmeas Lebesgue measurable subsets of and functions on $\mathbb{R}^n, T^n, S^n$
    - Lmeas bd of open Lebesgue measure of boundary of open sets in $\mathbb{R}^n$
    - met density Metric density of subsets of $\mathbb{R}^n$
    - Mobius n-sphere Mobius endomorphisms
    - monotone Monotone functions on $\mathbb{R}$
    - periodic int Cauchy Cauchy integral of periodic functions

- poly int Polygons with integer vertices
- R 2 open smooth End Open smooth maps $U \subseteq \mathbb{R}^2 \to \mathbb{C}$
- R n discrete subg Discrete subgroups of $\mathbb{R}^n$
- R n discrete subg cocpt Discrete cocompact subgroups of $\mathbb{R}^n$, flat tori
- RC ramified germs Ramified germs of smooth and holomorphic functions
- Rn open Open subsets of $\mathbb{R}^n$
- Rn open Riem Open subsets of $\mathbb{R}^n$ equipped with the flat metric
- smooth quasi-analytic Quasi-analytic smooth functions on $\mathbb{R}$
- star shaped Star-shaped subsets of $\mathbb{R}^n$
- Vec ODEs in $\mathbb{R}^n \leftrightarrow$ Vector fields in $\mathbb{R}^n$
- wave

$$\partial_t^2 + \sum_{i=1}^{n} v_i^2 \partial_{x_i}^2$$