Info

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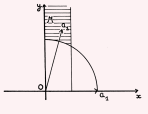
Discrete cocompact subgroups of $\mathbb{R}^n$

$\leftrightarrow$

flat tori

$n = 2$

quotients of $L := {GL(2)(\mathbb{R})}/{GL(}$	homeo	moduli of lattices/flat tori	$\mathbb{Z}$-basis
L	**4-manifold?**	**all lattices/tori**	
$\frac{L}{O_+(2,\mathbb{R})}$	$\mathbb{R}^3$	**upto oriented isometries**	
$\frac{L}{CO_+(2,\mathbb{R})} \cong? \frac{H}{SL(2)}$	$\mathbb{C} \cong \mathbb{R}^2$	**upto oriented conformal maps**	$v_1 = (1,0)$ **and** $v_2 \in F$
$\frac{L}{O(2,\mathbb{R})}$	$[0,\infty) \times \mathbb{R}^2$**?**	**upto isometries**	12-5. CLASSIFICATION OF FLAT TORI: Let $\mathbb{T}^2 = \mathbb{S}^1 \times \mathbb{S}^1$ denote the 2-torus. Show that if g is a flat Riemannian metric on $\mathbb{T}^2$, then $(\mathbb{T}^2, g)$ is isometric to one and only one Riemannian quotient $\mathbb{R}^2/\Lambda$, where Λ is a lattice generated by a basis of the form $((a,0),(b,c))$, with $a > 0$, $0 \le b \le a/2$, $c > 0$, and $b^2 + c^2 \ge a^2$. [Hint: Given a lattice $\Lambda \subseteq \mathbb{R}^2$, let v_1 be an element of $\Lambda \setminus \{0,0\}$ of minimal norm; let v_2 be an element of $\Lambda \setminus \{v_1\}$ of minimal norm (where $\{v_1\}$ is the cyclic subgroup generated by v_1), chosen so that the angle between v_1 and v_2 is less than or equal to $\pi/2$; and then apply a suitable orthogonal transformation.] [1]
$\frac{L}{CO(2,\mathbb{R})} \cong? \frac{H/SL(}{\mathbb{Z}_2}$	$[0,\infty) \times \mathbb{R}$**?**	**upto (unoriented) conformal maps**	$1/2$ (if not $a_2 - a_1$ would be shorter than a_2). The class of the lattice Γ is hence determined by the position of a_3 in the hachured domain $\mathcal{M} = \{(x,y)/x^2 + y^2 \ge 1, 0 \le x \le 1/2, y > 0\}$, and two lattices corresponding to two different points of $\mathcal{M}$ belong to two different classes. 

1. https://www.desmos.com/calculator/l6fgnougt6 ↩

Current note has 0 direct children and 0 total descendants.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - R n discrete subg cocpt Discrete cocompact subgroups of $\mathbb{R}^n$, flat tori

And it has 36 siblings.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - 1Hol Holomorphic functions on spaces over $\mathbb{C}$ of dimension 1
    - circle packing Circle packing on $\mathbb{R}^2$
    - circle packing to Riemann map Circle packing converges to the Riemann biholomorphism
    - Cn conn open bounded Bounded connected open subsets of $\mathbb{C}^n$
    - Cn conn open circular Connected circular open subsets of $\mathbb{C}^n$
    - cont Continuous functions on $\mathbb{R}^d$
    - cube dyadic Dyadic cubes
    - curves Curves
    - derivative Differentiable functions
    - forms Differential forms on $\mathbb{R}^n$
    - Fourier-Wigner Fourier-Wigner transform
    - harmonic composed conformal Harmonic functions composed with conformal maps
    - Hilbert Hilbert transform
    - hol harmonic disk-circle Fourier-Cauchy-Poisson correspondence of holomorphic and harmonic functions on the unit disk and their boundary values
    - Hol sets Holomorphic subsets of $\mathbb{C}^n$
    - hypersurf 2n reg Regular hypersurfaces in $\mathbb{R}^{2n}$
    - hypersurf or Orientable hypersurfaces in $\mathbb{R}^n$
    - KG

$$\partial_t^2 + \sum_{i=1}^{n} v_i^2 \partial_{x_i}^2 + m^2$$

    - Laplace Laplace operator on $\mathbb{R}^n$
    - Lmeas Lebesgue measurable subsets of and functions on $\mathbb{R}^n, T^n, S^n$
    - Lmeas bd of open Lebesgue measure of boundary of open sets in $\mathbb{R}^n$
    - met density Metric density of subsets of $\mathbb{R}^n$
    - Mobius n-sphere Mobius endomorphisms
    - monotone Monotone functions on $\mathbb{R}$

- periodic int Cauchy Cauchy integral of periodic functions
- poly int Polygons with integer vertices
- R 2 open smooth End Open smooth maps $U \subseteq \mathbb{R}^2 \to \mathbb{C}$
- R n discrete subg Discrete subgroups of $\mathbb{R}^n$
- R n discrete subg cocpt Discrete cocompact subgroups of $\mathbb{R}^n$, flat tori
- RC ramified germs Ramified germs of smooth and holomorphic functions
- Rn open Open subsets of $\mathbb{R}^n$
- Rn open Riem Open subsets of $\mathbb{R}^n$ equipped with the flat metric
- smooth quasi-analytic Quasi-analytic smooth functions on $\mathbb{R}$
- star shaped Star-shaped subsets of $\mathbb{R}^n$
- Vec ODEs in $\mathbb{R}^n \leftrightarrow$ Vector fields in $\mathbb{R}^n$
- wave

$$\partial_t^2 + \sum_{i=1}^{n} v_i^2 \partial_{x_i}^2$$