Info

This note found here
as a part of a collection
is written (completely with human hands) by Rupadarshi Ray,
created on February 2, 2026 10:16:59 PM,
and was last modified on February 2, 2026 10:17:42 PM.

Open subsets of $\mathbb{R}^n$ equipped with the flat metric

Weyl's asymptotic law

Let $\Omega \subset \mathbb{R}^n$ is open with piecewise smooth boundary and with Laplacian $\Delta = -\sum_{i=1}^{n} \frac{\partial^2}{\partial x_i^2}$ let

$$\begin{aligned} \Delta\varphi &= \lambda\varphi \\ \varphi|_{\partial\Omega} &= 0 \end{aligned}$$

be the Helmholtz equation with Dirichlet boundary condition.

- **the spectrum**

$$\begin{aligned} \operatorname{spec}(\Omega) &:= \sigma_{\mathsf{p}}(\Delta : L^2(\Omega) \to L^2(\Omega)) \\ &= \{\lambda \mid \Delta\varphi = \lambda\varphi\} \end{aligned}$$

  is known to be discrete with

$$0 < \lambda_1 \le \lambda_2 \le \cdots \to \infty$$

- **the eigenfunctions of Δ form an orthonormal basis of $L^2(\Omega)$ (when normalized)**

(Weyl's asymptotic law, 1911) Let $\Omega \subset \mathbb{R}^n$ is open with piecewise smooth boundary. Then element λ_k of spectrum of Ω grow like

$$\lambda_k \sim \frac{(2\pi)^2}{(m(D^n)m(\Omega))^{2/n}} k^{2/n} \text{ as } k \to \infty$$

So the spectrum determines the volume!

Definition. Isospectral open sets of $\mathbb{R}^n$

Two open sets $\Omega_1, \Omega_2 \subset \mathbb{R}^n$ are called isospectral if

$$\operatorname{spec}(\Omega_1) = \operatorname{spec}(\Omega_2)$$

and the multiplicity of each eigenvalue are also equal, that is for each $\lambda \in \operatorname{spec}(\Omega_1) = \operatorname{spec}(\Omega_2)$ we have

$$\dim\ker(\Delta\Big|_{L^2(\Omega_1)} - \lambda \mathrm{Id}) = \dim\ker(\Delta\Big|_{L^2(\Omega_2)} - \lambda \mathrm{Id})$$

Proposition: Isometric open sets are isospectral!

Current note has 0 direct children and 0 total descendants.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - Rn open Riem Open subsets of $\mathbb{R}^n$ equipped with the flat metric

And it has 36 siblings.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - 1Hol Holomorphic functions on spaces over $\mathbb{C}$ of dimension 1
    - circle packing Circle packing on $\mathbb{R}^2$
    - circle packing to Riemann map Circle packing converges to the Riemann biholomorphism
    - Cn conn open bounded Bounded connected open subsets of $\mathbb{C}^n$
    - Cn conn open circular Connected circular open subsets of $\mathbb{C}^n$
    - cont Continuous functions on $\mathbb{R}^d$
    - cube dyadic Dyadic cubes
    - curves Curves
    - derivative Differentiable functions
    - forms Differential forms on $\mathbb{R}^n$
    - Fourier-Wigner Fourier-Wigner transform
    - harmonic composed conformal Harmonic functions composed with conformal maps
    - Hilbert Hilbert transform
    - hol harmonic disk-circle Fourier-Cauchy-Poisson correspondence of holomorphic and harmonic functions on the unit disk and their boundary values
    - Hol sets Holomorphic subsets of $\mathbb{C}^n$

- hypersurf 2n reg Regular hypersurfaces in $\mathbb{R}^{2n}$
- hypersurf or Orientable hypersurfaces in $\mathbb{R}^n$
- KG

$$\partial_t^2 + \sum_{i=1}^{n} v_i^2 \partial_{x_i}^2 + m^2$$

- Laplace Laplace operator on $\mathbb{R}^n$
- Lmeas Lebesgue measurable subsets of and functions on $\mathbb{R}^n, T^n, S^n$
- Lmeas bd of open Lebesgue measure of boundary of open sets in $\mathbb{R}^n$
- met density Metric density of subsets of $\mathbb{R}^n$
- Mobius n-sphere Mobius endomorphisms
- monotone Monotone functions on $\mathbb{R}$
- periodic int Cauchy Cauchy integral of periodic functions
- poly int Polygons with integer vertices
- R 2 open smooth End Open smooth maps $U \subseteq \mathbb{R}^2 \to \mathbb{C}$
- R n discrete subg Discrete subgroups of $\mathbb{R}^n$
- R n discrete subg cocpt Discrete cocompact subgroups of $\mathbb{R}^n$, flat tori
- RC ramified germs Ramified germs of smooth and holomorphic functions
- Rn open Open subsets of $\mathbb{R}^n$
- Rn open Riem Open subsets of $\mathbb{R}^n$ equipped with the flat metric
- smooth quasi-analytic Quasi-analytic smooth functions on $\mathbb{R}$
- star shaped Star-shaped subsets of $\mathbb{R}^n$
- Vec ODEs in $\mathbb{R}^n \leftrightarrow$ Vector fields in $\mathbb{R}^n$
- wave

$$\partial_t^2 + \sum_{i=1}^{n} v_i^2 \partial_{x_i}^2$$