

Info

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Hamiltonian vector fields in \mathbb{R}^{2n} with standard symplectic form

Definition. Hamiltonian vector fields in \mathbb{R}^{2n} with standard symplectic form

For $H \in \mathcal{C}^1(\mathbb{R}^{2n})$, the hamiltonian vector field in \mathbb{R}^{2n} with coordinates (q_i, p_i) is

$$\begin{aligned}\dot{q} &= \frac{\partial H}{\partial p} \\ \dot{p} &= -\frac{\partial H}{\partial q}\end{aligned}$$

which we can write as

$$\begin{bmatrix} \dot{p} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial p} \\ \frac{\partial H}{\partial q} \end{bmatrix}$$

which on \mathbb{C}

$$[\dot{p} + i\dot{q}] = i \left[\frac{\partial H}{\partial p} + i \frac{\partial H}{\partial q} \right]$$

On \mathbb{C}^n , making $z_i := p + iq$ this becomes

$$\begin{aligned}[\dot{z}_i] &= \begin{bmatrix} i & & & \\ & i & & \\ & & i & \\ & & \dots & \\ & & & i \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial p_i} + i \frac{\partial H}{\partial q_i} \\ \\ \\ \end{bmatrix} \\ \dot{z}_i &= i \left(\frac{\partial H}{\partial p_i} + i \frac{\partial H}{\partial q_i} \right)\end{aligned}$$

or

$$\dot{z}_i = 2i \frac{\partial H}{\partial \bar{z}_i}$$

more compactly.

On $n = 1$, we have

$$\frac{\dot{z}}{2i} = \frac{\partial H}{\partial \bar{z}}$$

where

$$\frac{\partial H}{\partial \bar{z}} : \mathbb{C} \rightarrow \mathbb{C}$$

which

$$\frac{\partial}{\partial \bar{z}} \frac{\partial H}{\partial \bar{z}} = \frac{1}{2} \left(\frac{\partial}{\partial p_i} + i \frac{\partial}{\partial q_i} \right) \frac{1}{2} \left(\frac{\partial H}{\partial p_i} + i \frac{\partial H}{\partial q_i} \right) = \frac{1}{4}$$

- Canonical quantization will give stock.SPD.rep Schr-H
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 - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
 - Vec ODEs in $\mathbb{R}^n \leftrightarrow$ Vector fields in \mathbb{R}^n
 - Hamiltonian Hamiltonian vector fields in \mathbb{R}^{2n} with standard symplectic form

And it has 10 siblings.

- stamp stamp
 - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
 - Vec ODEs in $\mathbb{R}^n \leftrightarrow$ Vector fields in \mathbb{R}^n

- [cons](#) Constant of a flow in \mathbb{R}^n
- [Euler method](#) Euler's iterative solution method for first order ODEs
- [existence](#) Existence of integral curves of vector fields in \mathbb{R}^n
- [fixed](#) Fixed points
- [flows](#) Flow of a vector field in \mathbb{R}^n
- [from graphs](#) Graph \rightarrow polynomial ODE
- [grad](#) Gradient flows on \mathbb{R}^n
- [Hamiltonian](#) Hamiltonian vector fields in \mathbb{R}^{2n} with standard symplectic form
- [probability distribution](#) Probability distribution of flow
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