Info

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Flow of a vector field in $\mathbb{R}^n$

Also the standard 1-order ODE in $\mathbb{R}^n$, the standard *ordinary differential equation*.

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title: geometric pov on differential equations: $1$-ODE in $\R^{n}$



| solving differential equations                    | analysis and geometry of vector fields
|
| ------------------------------------------ | ----------------------------------- |
| an equation $$\dot{x}=\mathbf{f}(x)$$                                  | a vector field
$$\mathbf{f}:\mathcal{U} \subseteq \R^{n} \to \R^{n}$$        |
| solutions of the equation $$\begin{align} \dot{x}(t)&=\mathbf{f}(x(t)) \\ x(0)&=x_0
\end{align}$$                | integral curves of the vector field $$t \mapsto
\Phi^\mathbf{f}_{t}(x_{0})$$ |
| how solutions depend on initial conditions $$x_0 \mapsto x(t)$$ | flows of the vector field
$$x_0 \mapsto \Phi^\mathbf{f}_{t}(x_0)$$                          |
| conserved quantities                          | integrals of the vector field      |
| (linearly) decoupling the differential equation $$\dot{z_{i}}= f_{i}(z_i)$$ | (linear)
coordinate transformation such that $$\mathbf{f}=\sum_i f_i(z_{i}) \hat{z}_i$$ |
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Definition. Flow of a vector field

Given a vector field $\mathbf{f}:\mathcal{P}\subseteq\mathbb{R}^n\to\mathbb{R}^n$ where $\mathcal{P}$ is called the *phase space*, a map $\Phi:\mathcal{D}\subseteq\mathbb{R}\times\mathcal{P}\to\mathbb{R}^n$

$$(t,\mathbf{x})\mapsto\Phi_t(\mathbf{x})$$

is the **flow** of $\mathbf{f}$ if

$$\frac{\mathrm{d}}{\mathrm{d}t}\Phi_t(x_0)=f(\Phi_t(x_0))$$

pov on differential equations: ***convert n-degree differential equation in $\mathbb{R}$ to 1 -ODE in $\mathbb{R}^n$*** >

From $f : \mathbb{R} \to \mathbb{R}$

$$\mathfrak{D}^n f(x) = \mathcal{F}(x, f(x), \mathfrak{D} f(x), \dots, \mathfrak{D}^{n-1} f(x))$$

$$\downarrow$$

$q \in \mathbb{R}^{n+1}$ $q_i(x) := \mathfrak{D} f^n(x)$

$$\frac{\mathrm{d}}{\mathrm{d}t} \begin{pmatrix} q_0 \\ q_1 \\ \dots \\ q_n \end{pmatrix} = \begin{pmatrix} q_1 \\ q_2 \\ \dots \\ \mathcal{F}(q_0, q_1, \dots, q_{n-1}) \end{pmatrix}$$

Definition. **Local and global solutions**

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stamp.Rf.Vec.volume

- ~~cat~~
- t - types/extension
  - types of $\mathcal{D}$
  - types of f
    - linear
    - polynomial
      - quadratic
    - rational
  - types of Φ_f
    - conservative/dissipative
- p - constructing more/attachments/constructed with
  - from the standard n-ODE in $\mathbb{R}$
  - vector fields
  - from derivatives of $\mathbb{R} \to G$
- m - structures/more structure
  - integrals Conserved quantities for Flow of a vector field in $\mathbb{R}^n$
  - k-integrable

- symmetry

- v - properties space.R.n.f.field vector flows.v
  - local
    - existence, uniqueness
  - global
    - orbits
      - periodic
    - volumes/measure
    - stamp.Rf.Vec.probability distribution
  - parameters
    - bifurcations
- ~~s - subsets~~
- ~~f - functions/equations(polynomials, equations, differential equations)/dynamics~~
  - equivalent flows?
- b - function spaces, attachments, bundles and duals

- o - external effects
- e - equivalences
  - equivalences and automorphisms
  - identifications
- c - list/examples/classification upto isom

(Peano-Cauchy)

Let f be continuous in

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is the flow of f if

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then $\exists$ at least one local solution of the dynamical equation.

Every solution $x(t)$ of

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with $x(0) = x_0$ can be continued to a maximal level of existence $(t_1, t_2) \ni 0$. If t_1, t_2 are finite, then for any compact $\mathcal{K} \subseteq \mathcal{D}$, $\exists t \in (t_1, t_2)$ with $x(t) \notin \mathcal{K}$.

The last implication means that solutions will diverse or reach $\partial\mathcal{D}$.

Children

Dataview: No results to show for list query.

Current note has 0 direct children and 0 total descendants.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - Vec ODEs in $\mathbb{R}^n \leftrightarrow$ Vector fields in $\mathbb{R}^n$
      - flows Flow of a vector field in $\mathbb{R}^n$

And it has 10 siblings.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$

- Vec ODEs in $\mathbb{R}^n \leftrightarrow$ Vector fields in $\mathbb{R}^n$
  - cons Constant of a flow in $\mathbb{R}^n$
  - Euler method Euler's iterative solution method for first order ODEs
  - existence Existence of integral curves of vector fields in $\mathbb{R}^n$
  - fixed Fixed points
  - flows Flow of a vector field in $\mathbb{R}^n$
  - from graphs Graph $\to$ polynomial ODE
  - grad Gradient flows on $\mathbb{R}^n$
  - Hamiltonian Hamiltonian vector fields in $\mathbb{R}^{2n}$ with standard symplectic form
  - probability distribution Probability distribution of flow
  - volume Volume change by flows on $\mathbb{R}^n$