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$\mathcal{C}(U, X)$

Definition. $\mathcal{C}(U, X)$

Let (X, d) be a complete metric space, $U \subseteq \mathbb{R}^n$ be a open subset and

$$\mathbf{EndTop}(U, X)$$

be the space of all continuous functions $U \to X$.

- For $K \subseteq U$ compact we have the distance

$$\rho_K(f, g) := \sup \{d(f(z), g(z)) | z \in K\}$$

  for all $f, g \in \mathcal{C}(G, X)$.
- Then for $U = \bigcup_{n \in \mathbb{N}} K_n$ where K_n is compact and $K_n \subsetneq \operatorname{int}(K_{n+1})$ we define the metric

$$\rho(f, g) := \sum_{n \in \mathbb{N}} \frac{1}{2^n} \frac{\rho_{K_n}(f, g)}{1 + \rho_{K_n}(f, g)}$$

  on $\mathbf{EndTop}(U, X)$.

The (metrizable) topological space $\mathbf{EndTop}(U, X)$ with the topology induced by the metric ρ is denoted $\mathcal{C}(U, X)$.

Proposition: Consider the space equipped with the distance

$$(\mathbf{EndTop}(U, X), \rho)$$

- ρ is a complete metric

- the topology is given by

Definition. Compact-open topology on Y^X

Let X be a topological space and (Y,d) be a metric space. Let $K \subseteq X$ be compact and $\epsilon > 0$ and consider the subsets

$$B_\epsilon(f,K) := \left\{ g \in Y^X \,\middle|\, \sup_{x \in K} d(f(x), g(x)) < \epsilon \right\}$$

The topology on Y^X generated by these subsets is the **compact-open** topology $\mathfrak{T}_{\text{cpt}}$.

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- stamp stamp
    - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
        - cont Continuous functions on $\mathbb{R}^d$
            - space $\mathcal{C}(U,X)$

And it has 5 siblings.

- stamp stamp
    - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
        - cont Continuous functions on $\mathbb{R}^d$
            - 0to1 w bd $\mathcal{C}_0[0,1]$
            - 1with inf limit is unf Continuous functions on $\mathbb{R}$ with infinite limit are uniformly continuous
            - interm Intermedia value property of continuous functions on intervals
            - space $\mathcal{C}(U,X)$
            - space cpt $\mathcal{C}[0,1]$