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Dyadic cubes

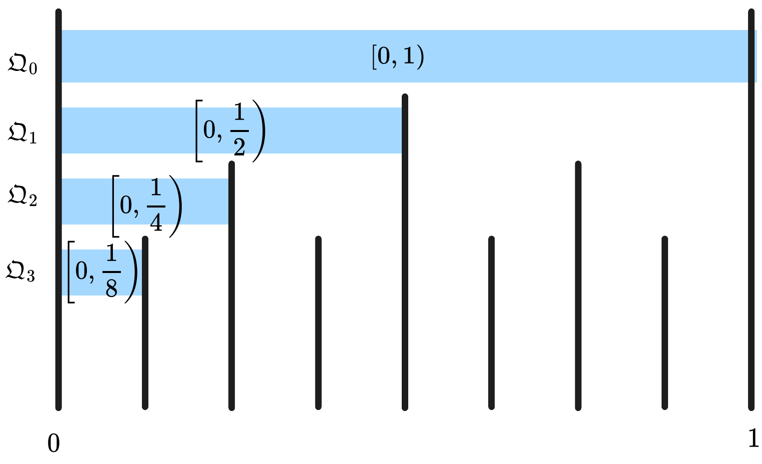
Definition. Dyadic cubes

A **dyadic cube** in $\mathbb{R}^d$ of *generation $n \in \mathbb{Z}$* is an element of

$$\mathfrak{Q}_n := \left\{2^n (k + [0,1)^d \middle| k \in \mathbb{Z}^d\right\}$$

$d = 1$

$$\mathfrak{Q}_0 = \{\ldots, [-1,0), [0,1), [1,2), [2,3), \ldots\}$$
$$\mathfrak{Q}_{-1} = \left\{\ldots, \frac{1}{2}[-1,0), \frac{1}{2}[0,1), \frac{1}{2}[1,2), \frac{1}{2}[2,3), \ldots\right\}$$



Therefore, dyadic cubes of generation $-m$, $\mathfrak{Q}_{-m}$, for $m \geq 1$ partitions $0 \in \mathfrak{Q}_0$ into 2^m subsets

$$\left[0, \frac{1}{2^m}\right), \left[\frac{1}{2^m}, \frac{2}{2^m}\right), \ldots, \left[\frac{2^m - 1}{2^m}, 1\right)$$

partition using dyadic cubes

Proposition:

$$\mathbb{R}^d = \bigsqcup \mathfrak{Q}_n$$

Proposition: Let Q_1 and Q_2 be two dyadic cubes that intersect $Q_1 \cap Q_2 \neq \emptyset$. Then either $Q_1 \subseteq Q_2$ or $Q_2 \subseteq Q_1$.

covering lemma

(Covering lemma for dyadic cubes) Let $Q_1, \dots, Q_N$ be a finite collection of dyadic cubes. Then there is a subcollection $Q_{n_1}, \dots, Q_{n_M}$ of disjoint dyadic cubes such that

$$Q_{n_1} \sqcup \cdots \sqcup Q_{n_M} = Q_1 \cup \cdots \cup Q_N$$

Consider the maximal dyadic cubes in the collection

$$\{Q_{n_1}, \dots, Q_{n_M}\} := \max(\{Q_1, \dots, Q_N\}, \subseteq)$$

- By

  Proposition: Let Q_1 and Q_2 be two dyadic cubes that intersect $Q_1 \cap Q_2 \neq \emptyset$. Then either $Q_1 \subseteq Q_2$ or $Q_2 \subseteq Q_1$.

  either Q_{n_i} are all disjoint or they are not maximal.

σ-algebra

Definition. **Let**

$$\langle \mathfrak{Q}_n \rangle$$

be the σ-algebra generated by dyadic cubes of generation n.

Proposition:

$$\langle \mathfrak{Q}_n \rangle = \{\text{countable union of elements in } \mathfrak{Q}_n\}$$

Proposition: A function $f : \mathbb{R}^d \to \mathbb{C}$ is $\langle \mathfrak{Q}_n \rangle$-measurable $\iff$ f is constant on each dyadic cube in $\mathfrak{Q}_n$.

Because

$$\langle \mathfrak{Q}_n \rangle \subset \mathfrak{B}_{\mathbb{R}^d}$$

we have

$$L^2(\mathbb{R}^d, \langle \mathfrak{Q}_n \rangle, m) \hookrightarrow L^2(\mathbb{R}^d, \mathfrak{B}_{\mathbb{R}^d}, m)$$

is an isometric embedding. This is of course simply the inclusion of functions constant on each dyadic cube of generation n.

Its adjoint, the conditional expectation is averaging over each dyadic cube:

$$\boldsymbol{E}(f | \langle \mathfrak{Q}_n \rangle)(x) = \begin{cases} \frac{1}{m(Q)} \int_Q f & x \in Q \in \mathfrak{Q}_n \end{cases}$$

Current note has 0 direct children and 0 total descendants.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - cube dyadic Dyadic cubes

And it has 36 siblings.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - 1Hol Holomorphic functions on spaces over $\mathbb{C}$ of dimension 1
    - circle packing Circle packing on $\mathbb{R}^2$
    - circle packing to Riemann map Circle packing converges to the Riemann biholomorphism
    - Cn conn open bounded Bounded connected open subsets of $\mathbb{C}^n$
    - Cn conn open circular Connected circular open subsets of $\mathbb{C}^n$
    - cont Continuous functions on $\mathbb{R}^d$
    - cube dyadic Dyadic cubes
    - curves Curves
    - derivative Differentiable functions

- forms Differential forms on $\mathbb{R}^n$
- Fourier-Wigner Fourier-Wigner transform
- harmonic composed conformal Harmonic functions composed with conformal maps
- Hilbert Hilbert transform
- hol harmonic disk-circle Fourier-Cauchy-Poisson correspondence of holomorphic and harmonic functions on the unit disk and their boundary values
- Hol sets Holomorphic subsets of $\mathbb{C}^n$
- hypersurf 2n reg Regular hypersurfaces in $\mathbb{R}^{2n}$
- hypersurf or Orientable hypersurfaces in $\mathbb{R}^n$
- KG

$$\partial_t^2 + \sum_{i=1}^{n} v_i^2 \partial_{x_i}^2 + m^2$$

- Laplace Laplace operator on $\mathbb{R}^n$
- Lmeas Lebesgue measurable subsets of and functions on $\mathbb{R}^n, T^n, S^n$
- Lmeas bd of open Lebesgue measure of boundary of open sets in $\mathbb{R}^n$
- met density Metric density of subsets of $\mathbb{R}^n$
- Mobius n-sphere Mobius endomorphisms
- monotone Monotone functions on $\mathbb{R}$
- periodic int Cauchy Cauchy integral of periodic functions
- poly int Polygons with integer vertices
- R 2 open smooth End Open smooth maps $U \subseteq \mathbb{R}^2 \to \mathbb{C}$
- R n discrete subg Discrete subgroups of $\mathbb{R}^n$
- R n discrete subg cocpt Discrete cocompact subgroups of $\mathbb{R}^n$, flat tori
- RC ramified germs Ramified germs of smooth and holomorphic functions
- Rn open Open subsets of $\mathbb{R}^n$
- Rn open Riem Open subsets of $\mathbb{R}^n$ equipped with the flat metric
- smooth quasi-analytic Quasi-analytic smooth functions on $\mathbb{R}$
- star shaped Star-shaped subsets of $\mathbb{R}^n$
- Vec ODEs in $\mathbb{R}^n$ $\leftrightarrow$ Vector fields in $\mathbb{R}^n$
- wave

$$\partial_t^2 + \sum_{i=1}^{n} v_i^2 \partial_{x_i}^2$$