Info

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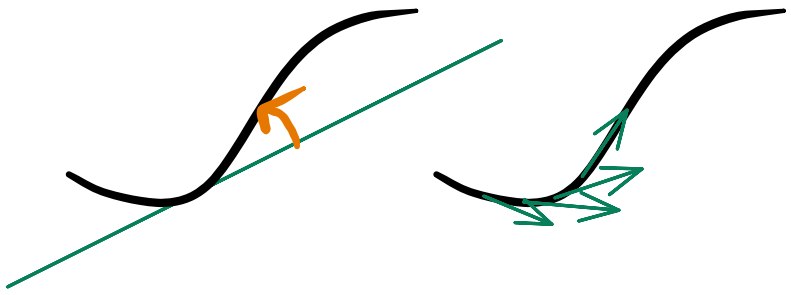
Curvature of a curve

Motivation: "How much does a curve *curve*?"



be "measured" by how much the unit tangent vector changes with t.

Definition. **Curvature $\kappa(t)$ of a curve**

Given a *unit speed parameterization* of a curve $\tilde{\gamma}$, the

$$\kappa[\tilde{\gamma}](t) := \left\|\tilde{\gamma}''(t)\right\|$$

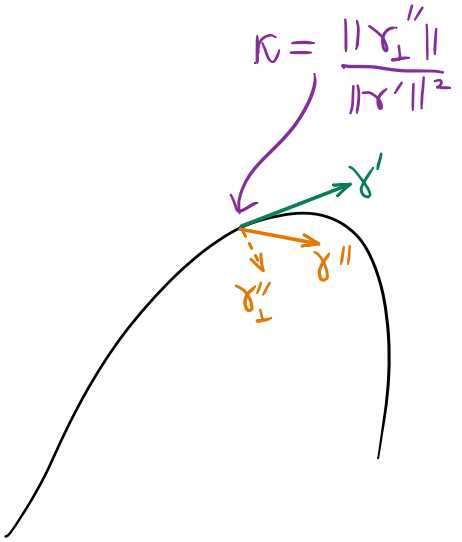
is the **curvature** of the curve at the point $\gamma(t)$.

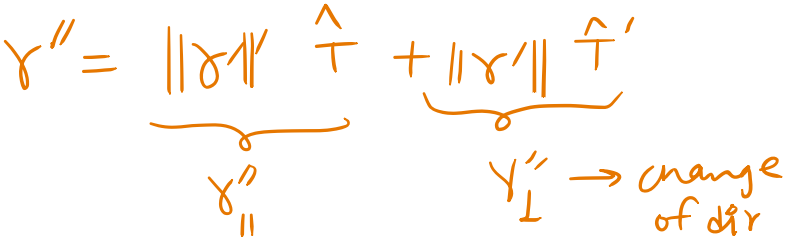
Between any two unit-speed parameterizations, as acceleration stays invariant, curvature also remains invariant.

For any parameterization γ the curvature is

$$\kappa[\gamma](t) = \frac{\left\|\gamma'' - (\gamma'' \cdot \hat{\mathbf{T}})\hat{\mathbf{T}}\right\|}{\left\|\gamma'\right\|^2}(t)$$

The $\gamma'' - (\gamma'' \cdot \hat{\mathbf{T}})\hat{\mathbf{T}}$ term just takes the acceleration and removes the component of it parallel to velocity. Thus the curvature is just the norm of the component of acceleration perpendicular to velocity, divided by norm of velocity squared.





Under any reparameterization, the curvature should remain invariant, because it is defined from the unit-speed parameterization, and it actually is:

$$\kappa[\gamma \circ \theta] = \kappa[\gamma] \circ \theta$$

Current note has 0 direct children and 0 total descendants.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - curves Curves
      - curvature Curvature of a curve

And it has 6 siblings.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - curves Curves

- arc Arc length
- congruence Congruent curves
- curvature Curvature of a curve
- reparameterization Reparameterization of a parameterized curve
- tangent Tangent of a curve
- unit speed Unit speed parameterization of a curve