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## Tangent of a curve

### Definition. Tangent unit of a curve

Given a *regular* parameterized curve  $\gamma$ , the **tangent unit** at the point  $\gamma(t)$  is

$$\hat{\mathbf{T}}[\gamma](t) := \frac{\gamma'}{\|\gamma'\|}$$

## reparameterizations

And it transforms under a reparameterization like:



$$\hat{\mathbf{T}}[\gamma \circ \theta] = \alpha_O[\theta] (\hat{\mathbf{T}}[\gamma] \circ \theta)$$



but it still points to the same “direction” - tangent to the curve - so we say *it remains invariant* under reparameterizations.

## Tangent line

### Definition. Tangent line of a curve

Given a *regular* parameterized curve  $\gamma$ , the **tangent unit** at the point  $\gamma(t)$  is

$$\mathbf{T}_L[\gamma](t) := \frac{\gamma'}{\|\gamma'\|}$$

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