

## Info

This note [found here](#)  
as a part of [a collection](#)  
is written (completely with human hands) by [Rupadarshi Ray](#),  
created on April 27, 2026 2:01:44 PM,  
and was last modified on April 27, 2026 2:07:30 PM.

## Basis of functions on $[0, 1], S^1$

**Proposition:** The  $\mathbb{C}$ -span of the following are dense in the corresponding spaces

- $\{x^n | n \geq 1\} \subseteq L^2[0, 1]$
- $\{z^n | n \in \mathbb{Z}\} \subseteq L^2(S^1)$
- $\{1\} \cup \{\sin(2n\pi x) | n \geq 1\} \cup \{\cos(2n\pi x) | n \geq 1\}$
- $\{\sin(n\pi x) | n \geq 1\}$
- $\{\cos(n\pi x) | n \geq 1\}$

Interestingly, this implies, *even* functions in  $L^2[0, 1]$  are dense!  
Similarly, odd functions.

---

Current note has 0 direct children and 0 total descendants.

- [stamp](#) stamp
  - [Rf](#) subobjects of and functions on  $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$ 
    - [dense](#)
      - [on 0to1](#) Basis of functions on  $[0, 1], S^1$

And it has 2 siblings.

- [stamp](#) stamp
  - [Rf](#) subobjects of and functions on  $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$ 
    - [dense](#)
      - [nowhere diffable in C](#) Nowhere differentiable functions are dense in  $\mathcal{C}[0, 1]$
      - [on 0to1](#) Basis of functions on  $[0, 1], S^1$