

### Info

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created on June 11, 2026 2:53:51 PM,  
and was last modified on June 11, 2026 2:57:04 PM.

## Fractional Sobolev spaces

### Definition. Definition

For any  $s \in \mathbb{R}$  the function

$$\xi \mapsto (1 + |\xi|^2)^{s/2} \in C^\infty(\mathbb{R}^d)$$

and is *slowly increasing*.

The map

$$\begin{aligned} \Lambda_s : \mathcal{S}^\star &\rightarrow \mathcal{S}^\star \\ f &\mapsto ((1 + |\xi|^2)^{s/2} \hat{f})^\vee \end{aligned}$$

is a linear homeomorphism with inverse  $\Lambda_{-s}$ .

### Definition. Definition

$$H^s := \{f \in \mathcal{S}^\star \mid \Lambda_s f \in L^2\}$$

[1]([https://rupadarshiray.github.io/notes/wiki.\\_Gerald\\_B\\_Folland\\_-\\_Real\\_Analysis\\_Modern\\_Techniques\\_and\\_Their\\_Applications\\_2nd\\_edition\\_\(1999\).pdf#page=318.pdf](https://rupadarshiray.github.io/notes/wiki._Gerald_B_Folland_-_Real_Analysis_Modern_Techniques_and_Their_Applications_2nd_edition_(1999).pdf#page=318.pdf))]

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1. Gerald B Folland - Real Analysis Modern Techniques and Their Applications 2nd edition (1999), p.318

