

 Info

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Double derivative/Laplace operator on the circle

We identify


$$\begin{aligned}\mathbb{R} &\rightarrow S^1 \\ x &\mapsto e^{2\pi i n x}\end{aligned}$$

so

$$L^p[0, 1] = L^p(S^1)$$

and

$$\{f \in C^k[0, 1] \mid f(0) = f(1)\} = C^k(S^1)$$

 **Definition.** Space of continuously double differentiable functions on S^1 with boundary value 0

$$\begin{aligned}\mathcal{D}^2 : C^2(S^1) &\rightarrow C^0(S^1) \\ f &\mapsto f''\end{aligned}$$

Proposition: The set of functions

$$\{e^{2\pi i n x} \mid n \in \mathbb{Z}\}$$

is a set of orthogonal basis of $L^2[0, 1]$.

