Info

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Infinite limit of derivatives

if infinite limits of f, f' exist then $f' \to 0$

Intuition

If f is a $\mathcal{C}^1[a,\infty)$ curve on $\mathbb{R}$ such that it reaches $f(x) \xrightarrow{x\to\infty} f(\infty)$ then it should be the case that f goes *slower* and *slower* as $x \to \infty$. So $f'(x) \xrightarrow{x\to\infty} 0$.

This however this is not true as if limit of $f'(x)$ does not exist as $x \to \infty$

Examples: functions on $[1,\infty)$ whose $x \to \infty$ limit is 0 but their derivatives oscillate or blows up.

	its derivative
$\frac{\cos x}{x}$	f(x)= \frac{\cos x}{x} f'(x)
$\frac{\cos x^2}{x}$	f(x)= \frac{\cos x^{2}}{x} f'(x)
$\frac{\cos x^3}{x}$	• $\frac{\mathrm{d}}{\mathrm{d}x}\frac{\cos x^3}{x} = -3x\sin(x^3) - \frac{\cos(x^3)}{x^2}$
\frac{\cos x^{3}}{x} 1/x	\frac{\cos x^{3}}{x} -3x \sin(x^{3}) - \frac{\cos(x^{3})}{x^{2}}

However, if $f'(x) \xrightarrow{x\to\infty} f'(\infty)$ exists then the intuitive claim is right!

Suppose for $a \in \mathbb{R}$ we have continuous $f, f' : [a, \infty) \to \mathbb{R}$ such that the limits

$$\lim_{x\to\infty} f(x), \lim_{x\to\infty} f'(x) \in \mathbb{R}$$

exist. Then

$$\lim_{x\to\infty} f'(x) = 0$$

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{e^x f(x)}{e^x} = \lim_{x\to\infty} (f(x) + f'(x))$$

For example:

- When γ

  $$\gamma' = F(\gamma)$$

  is **solution of a autonomous continuous vector field** $F : [a, \infty) \to \mathbb{R}$ such that γ has a $t \to \infty$ limit $\gamma(\infty) \in [a, \infty)$, then we also have

  $$\gamma'(t) = F(\gamma(t)) \xrightarrow{t\to\infty} F(\gamma(\infty))$$

  limit,

  - this is where the above theorem implies

    $$\gamma'(\infty) = F(\gamma(\infty)) = 0$$

  - So the (future) *endpoint* of the solution (if it exists, when it is a future-global solution) has to be a *fixed point* of the autonomous system!

Current note has 0 direct children and 0 total descendants.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - derivative Differentiable functions
      - limit infinite Infinite limit of derivatives

And it has 10 siblings.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - derivative Differentiable functions
      - 1 at a point Functions $(a, b) \to \mathbb{R}$ differentiable at a point
      - bd Comparing a function and it derivative
      - cont abs Absolutely continuous functions on $[a, b] \leftrightarrow \int_{[a,-]}(L^1[a, b])$
      - dist Distributional derivatives
      - double circle Double derivative/Laplace operator on the circle
      - frac Fractional derivative
      - limit infinite Infinite limit of derivatives
      - space Space of continuous and continuously differentiable functions on $\mathbb{R}$
      - total Derivative of maps $\mathbb{R}^n \to \mathbb{R}^m$
      - zoom Zooming of a map $\mathbb{R}^n \to \mathbb{R}^m$