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Differentiable functions

We have the *classical* way to differentiate functions.

Definition. Derivative of functions on $\mathbb{R}$

Let $f : I(\text{interval}) \subseteq \mathbb{R} \to \mathbb{R}$ be a function and $x \in I$. Then

$$\begin{aligned} I \setminus \{x\} &\to \mathbb{R} \\ t &\mapsto \frac{f(t) - f(x)}{t - x} \end{aligned}$$

is well-defined. Then $f'(x) \in \mathbb{R} \cup \{\infty, -\infty\}$ is defined to be

$$f'(x) := \lim_{(a,b) \setminus \{x\} \ni t \to x} \frac{f(t) - f(x)}{t - x}$$

whenever the right side exists in $\mathbb{R} \cup \{\infty, -\infty\}$.
The function f is called **differentiable** at p if $f'(p) \in \mathbb{R}$.
Higher derivatives are defined iteratively

$$\begin{aligned} f^{(0)} &:= f \\ f^{(n)}(x) &:= (f^{(n-1)})'(x) \end{aligned}$$

whenever they exist.

This may generalize to: f is *weakly differentiable* on $[a, b]$ if there is a $\psi \in L^1[a, b]$ such that

$$f(x) = c_1 + \int_{[a,x]} \psi$$

This is equivalent to f being absolutely continuous.

This generalizes further to

Definition. Distributional derivatives of L^1_{loc} functions

Let $U(\text{open}) \subseteq \mathbb{R}^n$ be an open set and $f \in L^1_{\text{loc}}(U)$. Then $w \in L^1_{\text{loc}}$ is a **distributional derivative** of f in $\hat{x}_j$ direction

$$\overset{\int}{\partial}_j f = w \text{ if } \forall \varphi \in \mathcal{C}^\infty_c(U), \int_U w\varphi = -\int_U f \partial_j \varphi$$

Current note has 10 direct children and 18 total descendants.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - derivative Differentiable functions
      - 1 at a point Functions $(a, b) \to \mathbb{R}$ differentiable at a point
      - bd Comparing a function and it derivative
      - cont abs Absolutely continuous functions on $[a, b] \leftrightarrow \int_{[a,-]}(L^1[a, b])$
      - dist Distributional derivatives
        - H-frac Fractional Sobolev spaces
        - H1 $H^1(U)$
        - H1 loc H^1_{loc}
        - H1 loc R2 $H^1_{\text{loc}}(U)$ for $U \subseteq \mathbb{R}^2$
      - double circle Double derivative/Laplace operator on the circle
      - frac Fractional derivative
      - limit infinite Infinite limit of derivatives
      - space Space of continuous and continuously differentiable functions on $\mathbb{R}$
        - cpt 1 $\mathcal{C}^1([a, b], \mathbb{R})$
          - End d Derivative operator on $\mathcal{C}^1[a, b]$
          - sup norm $(\mathcal{C}^1([a, b], \mathbb{R}), \| \|_\infty)$
        - open with cpt supp k $\mathcal{C}^k_c((a, b), \mathbb{R})$
      - total Derivative of maps $\mathbb{R}^n \to \mathbb{R}^m$
      - zoom Zooming of a map $\mathbb{R}^n \to \mathbb{R}^m$

And it has 36 siblings.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - 1Hol Holomorphic functions on spaces over $\mathbb{C}$ of dimension 1
    - circle packing Circle packing on $\mathbb{R}^2$

- circle packing to Riemann map Circle packing converges to the Riemann biholomorphism
- Cn conn open bounded Bounded connected open subsets of $\mathbb{C}^n$
- Cn conn open circular Connected circular open subsets of $\mathbb{C}^n$
- cont Continuous functions on $\mathbb{R}^d$
- cube dyadic Dyadic cubes
- curves Curves
- derivative Differentiable functions
- forms Differential forms on $\mathbb{R}^n$
- Fourier-Wigner Fourier-Wigner transform
- harmonic composed conformal Harmonic functions composed with conformal maps
- Hilbert Hilbert transform
- hol harmonic disk-circle Fourier-Cauchy-Poisson correspondence of holomorphic and harmonic functions on the unit disk and their boundary values
- Hol sets Holomorphic subsets of $\mathbb{C}^n$
- hypersurf 2n reg Regular hypersurfaces in $\mathbb{R}^{2n}$
- hypersurf or Orientable hypersurfaces in $\mathbb{R}^n$
- KG

$$\partial_t^2 + \sum_{i=1}^{n} v_i^2 \partial_{x_i}^2 + m^2$$

- Laplace Laplace operator on $\mathbb{R}^n$
- Lmeas Lebesgue measurable subsets of and functions on $\mathbb{R}^n, T^n, S^n$
- Lmeas bd of open Lebesgue measure of boundary of open sets in $\mathbb{R}^n$
- met density Metric density of subsets of $\mathbb{R}^n$
- Mobius n-sphere Mobius endomorphisms
- monotone Monotone functions on $\mathbb{R}$
- periodic int Cauchy Cauchy integral of periodic functions
- poly int Polygons with integer vertices
- R 2 open smooth End Open smooth maps $U \subseteq \mathbb{R}^2 \to \mathbb{C}$
- R n discrete subg Discrete subgroups of $\mathbb{R}^n$
- R n discrete subg cocpt Discrete cocompact subgroups of $\mathbb{R}^n$, flat tori
- RC ramified germs Ramified germs of smooth and holomorphic functions
- Rn open Open subsets of $\mathbb{R}^n$
- Rn open Riem Open subsets of $\mathbb{R}^n$ equipped with the flat metric
- smooth quasi-analytic Quasi-analytic smooth functions on $\mathbb{R}$
- star shaped Star-shaped subsets of $\mathbb{R}^n$
- Vec ODEs in $\mathbb{R}^n \leftrightarrow$ Vector fields in $\mathbb{R}^n$

- wave

$$\partial_t^2 + \sum_{i=1}^{n} v_i^2 \partial_{x_i}^2$$