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created on January 1, 2025 8:33:32 PM,
and was last modified on May 17, 2026 7:36:48 PM.

Derivative operator on $\mathcal{C}^1[a,b]$

Definition. **The derivative operator**

$$\begin{aligned}\mathfrak{D} : \mathcal{C}^1[a,b] &\to \mathcal{C}[a,b] \\ f &\mapsto f'\end{aligned}$$

In the sum norm

$$\mathfrak{D} : (\mathcal{C}^1[a,b], \|\ \|_\infty^1) \to (\mathcal{C}[a,b], \|\ \|_\infty)$$

we have

$$\|\mathfrak{D}(f)\|_\infty = \|f'\|_\infty \leq \|f\|_\infty + \|f'\|_\infty$$

hence $\mathfrak{D}$ is **bounded** and

$$\|\mathfrak{D}\| \leq 1$$

However, for the sup norm it is **not bounded**.

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- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - derivative Differentiable functions
      - space Space of continuous and continuously differentiable functions on $\mathbb{R}$
        - cpt 1 $\mathcal{C}^1([a,b],\mathbb{R})$
          - End d Derivative operator on $\mathcal{C}^1[a,b]$

And it has 2 siblings.

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          - sup norm $(\mathcal{C}^1([a,b],\mathbb{R}), \|\|_\infty)$