Info

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is written (completely with human hands) by Rupadarshi Ray,
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$(\mathcal{C}^1([a,b],\mathbb{R}), \|\ \|_\infty)$

$$(\mathcal{C}^1([a,b],\mathbb{R}), \|\ \|_\infty) \leq (\mathcal{C}^0([a,b],\mathbb{R}), \|\ \|_\infty)$$

Space of continuously differentiable functions is dense in the space of continuous functions on $[a,b]$ in supremum norm:

$$(\mathcal{C}^1([a,b],\mathbb{R}), \|\ \|_\infty) \leq (\mathcal{C}^0([a,b],\mathbb{R}), \|\ \|_\infty)$$

is dense. Therefore $(\mathcal{C}^1([a,b],\mathbb{R}), \|\ \|_\infty)$ is not a Banach space.

Every continuous function $[0,1]$ is a uniform limit ($\|\ \|_\infty$-limit) of polynomials (Weierstrass approximation) and polynomials are continuously differentiable so

$$(\mathcal{C}^1([a,b],\mathbb{R}), \|\ \|_\infty) \leq (\mathcal{C}^0([a,b],\mathbb{R}), \|\ \|_\infty)$$

is **not closed** and actually is dense. Thus $\mathcal{C}^1$ is not Banach in this norm.

∎

Current note has 0 direct children and 0 total descendants.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - derivative Differentiable functions
      - space Space of continuous and continuously differentiable functions on $\mathbb{R}$
        - cpt 1 $\mathcal{C}^1([a,b],\mathbb{R})$
          - sup norm $(\mathcal{C}^1([a,b],\mathbb{R}), \|\ \|_\infty)$

And it has 2 siblings.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - derivative Differentiable functions
      - space Space of continuous and continuously differentiable functions on $\mathbb{R}$
        - cpt 1 $\mathcal{C}^1([a,b],\mathbb{R})$
          - End d Derivative operator on $\mathcal{C}^1[a,b]$
          - sup norm $(\mathcal{C}^1([a,b],\mathbb{R}), \|\ \|_\infty)$