Info

This note found here
as a part of a collection
is written (completely with human hands) by Rupadarshi Ray,
created on May 22, 2026 3:35:43 PM,
and was last modified on May 24, 2026 6:41:47 PM.

Zooming of a map $\mathbb{R}^n \to \mathbb{R}^m$

Intuition

Let

$$f : U \text{ (open)} \subseteq \mathbb{R}^n \to \mathbb{R}^m$$

be a map. Then the **derivative** of f at $p \in U$ is the linear map

$$\mathfrak{D}_p f : T_p\mathbb{R}^n \to T_p\mathbb{R}^m$$

such that

$$f(p+v) \approx f(p) + \mathfrak{D}_p f(v)$$

Definition. **Zooming at a point**

Let

$$f : U \text{ (open)} \subseteq \mathbb{R}^n \to \mathbb{R}^m$$

be a map. Then the **derivative** of f at $p \in U$ is the affine map

$$\mathfrak{Z}_p f : \mathbb{R}^n \to \mathbb{R}^m$$

such that

$$\begin{aligned} & \mathfrak{Z}_p(p+v) && = f(p) + \mathfrak{D}_p f(v) \\ \iff \quad & \mathfrak{Z}_p(x) && = f(p) + \mathfrak{D}_p f(x-p) \end{aligned}$$

where $\mathfrak{D}_p f$ is the derivative at the point

Definition. **Derivative of a map $U \subseteq \mathbb{R}^n \to \mathbb{R}^m$ at a point**

Let

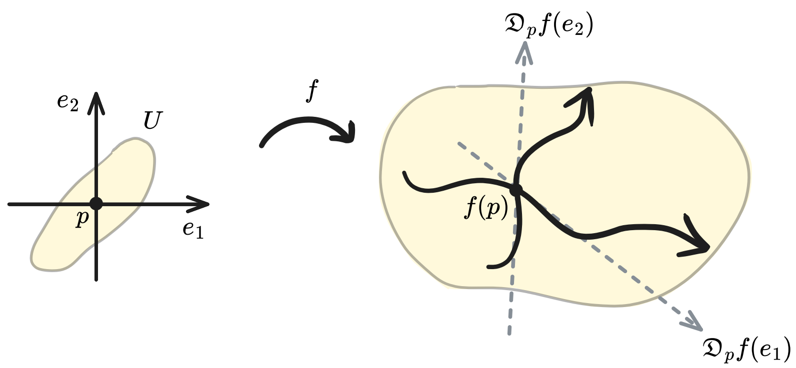
$$f : U \text{ (open)} \subseteq \mathbb{R}^n \to \mathbb{R}^m$$

be a map. Then the **derivative** of f at $p \in U$ is the linear map

$$\mathfrak{D}_p f : T_p\mathbb{R}^n \to T_p\mathbb{R}^m$$

such that

$$\lim_{v \to 0} \frac{f(p+v) - f(p) - \mathfrak{D}_p f(v)}{|v|} = 0$$
$$\iff \lim_{v \to 0} \frac{|f(p+v) - f(p) - \mathfrak{D}_p f(v)|}{|v|} = 0$$



$$\iff \lim_{x \to p} \frac{|f(x) - f(p) - \mathfrak{D}_p f(x-p)|}{|x-p|} = 0$$
$$\iff \forall \epsilon > 0 \exists r(\epsilon) > 0 :$$
$$\forall x \in B_{r(\epsilon)}(p), |f(x) - f(p) - \mathfrak{D}_p f(x-p)| \leq \epsilon |x-p|$$

Current note has 0 direct children and 0 total descendants.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - derivative Differentiable functions
      - zoom Zooming of a map $\mathbb{R}^n \to \mathbb{R}^m$

And it has 10 siblings.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$

- derivative Differentiable functions
  - 1 at a point Functions $(a, b) \to \mathbb{R}$ differentiable at a point
  - bd Comparing a function and it derivative
  - cont abs Absolutely continuous functions on $[a, b] \leftrightarrow \int_{[a,-]}(L^1[a, b])$
  - dist Distributional derivatives
  - double circle Double derivative/Laplace operator on the circle
  - frac Fractional derivative
  - limit infinite Infinite limit of derivatives
  - space Space of continuous and continuously differentiable functions on $\mathbb{R}$
  - total Derivative of maps $\mathbb{R}^n \to \mathbb{R}^m$
  - zoom Zooming of a map $\mathbb{R}^n \to \mathbb{R}^m$