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Fourier-Cauchy-Poisson correspondence of holomorphic and harmonic functions on the *unit disk* and their boundary value on the *unit circle*

when the boundary value is continuous on the unit circle

when the boundary value is L^2 on the unit circle

Proposition:

$$P : l^2(\mathbb{N}, \mathbb{C}) \to \mathit{Hardy}^2(D) \subset \mathcal{H}(D)$$
$$(a_n) \mapsto \sum_{n\geq 0} a_n z^n$$

$$F(z) := \sum_{n\geq 0} a_n z^n$$

For any $r < 1$

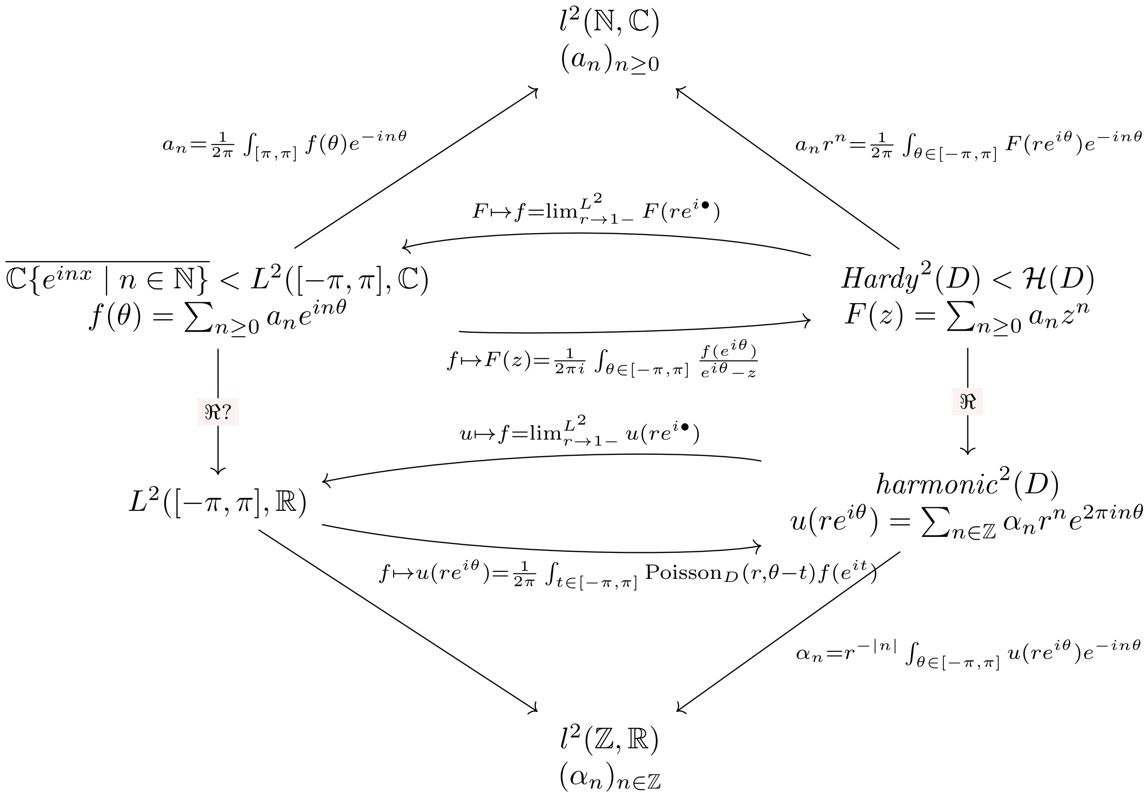
$$F(re^{i\theta}) = \sum_{n\geq 0} a_n r^n e^{i\pi n\theta}$$

where $a_n r^n$ is the Fourier coefficient of $f(\theta) := F(re^{i\theta})$ given by

$$a_n r^n = \frac{1}{2\pi} \int_{\theta\in[-\pi,\pi]} F(re^{i\theta})e^{-in\theta}$$

and the integral vanishes when $n < 0$.

$$\alpha_n = \begin{cases} a_0 & n = 0 \\ \frac{1}{2} a_n & n > 0 \\ \frac{1}{2} \bar{a}_{-n} & n < 0 \end{cases}$$



$$\Re\int_{S^1} \frac{f(e^{i\theta})}{e^{i\theta}-z}\mathrm{d}\theta$$

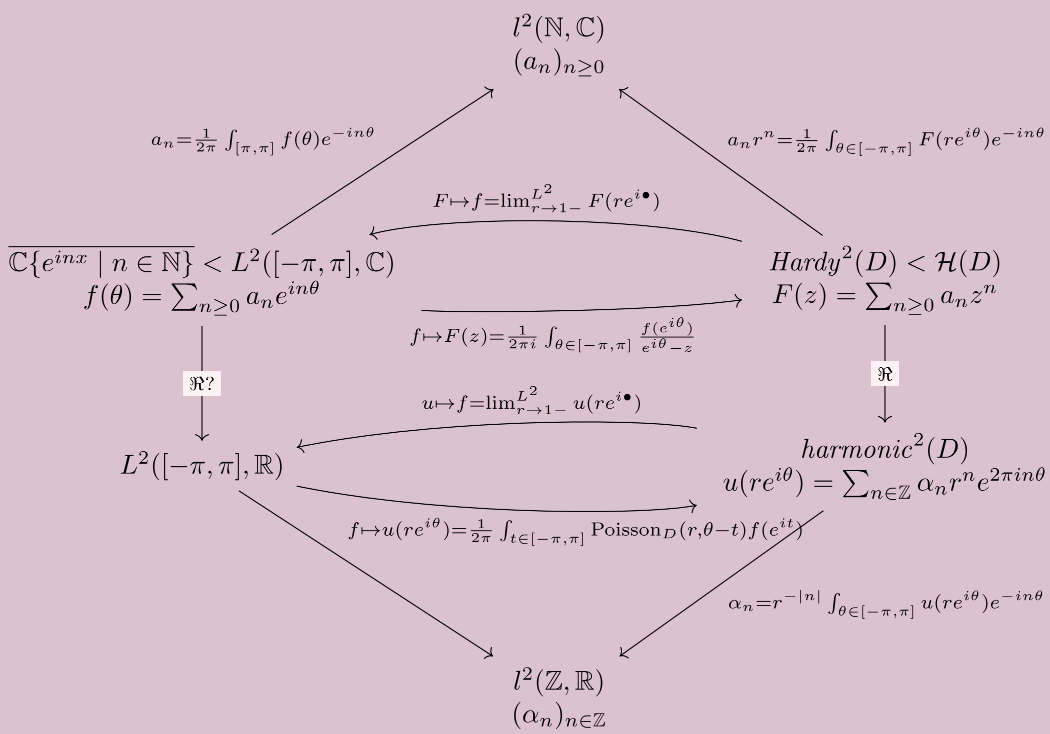
$$\sum_{n\geq 0} \hat{f}(n)z^n = \frac{1}{2\pi i}\int_{\theta\in[-\pi,\pi]} \frac{f(e^{i\theta})}{e^{i\theta}-z}$$

$$\begin{aligned} u(re^{it}) &= \Re\left(\frac{1}{2\pi i}\int_{\theta\in[-\pi,\pi]} \frac{f(e^{i\theta})}{e^{i\theta}-re^{it}}\right) \\ &= \frac{1}{2\pi}\int \Re\left(\frac{-i}{e^{i\theta}-re^{it}}\right)f(e^{i\theta}) \\ &= \frac{1}{2\pi}\int \Re\left(\frac{-ie^{-i\theta}}{1-re^{i(t-\theta)}}\right)f(e^{i\theta}) \end{aligned}$$

and here

$$\begin{aligned} \frac{1}{2\pi}\Re\left(\frac{-ie^{-i\theta}}{1-re^{i(t-\theta)}}\right) &= \frac{1}{2\pi}\Re\left(\frac{-ie^{-i\theta}(1-re^{-i(t-\theta)})}{(1-re^{i(t-\theta)})(1-re^{-i(t-\theta)})}\right) \\ &= \frac{1}{2\pi}\Re\left(\frac{i}{1+r^2-}\right) \end{aligned}$$

The diagram of $\mathbb{C}$-vector space endomorphisms of *Fourier* (left), *Fourier as well as Cauchy!* (top), *Poisson* (bottom)



commutes and each map is an unitary isomorphism of Hilbert spaces ?is the right morphism also unitary?

$$\frac{1}{i(e^{i\theta}-re^{it})}$$

$$\int_D |u|^2 < \infty$$

$$u(re^{i\theta}) = \int_{rz} \mathrm{d}u$$

$$u(re^{i\theta}) = \int_{ae^{i\theta}} X^\sharp$$

when boundary value is L^1

not unique [1]

Current note has 0 direct children and 0 total descendants.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$

- hol harmonic disk-circle Fourier-Cauchy-Poisson correspondence of holomorphic and harmonic functions on the unit disk and their boundary values

And it has 36 siblings.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - 1Hol Holomorphic functions on spaces over $\mathbb{C}$ of dimension 1
    - circle packing Circle packing on $\mathbb{R}^2$
    - circle packing to Riemann map Circle packing converges to the Riemann biholomorphism
    - Cn conn open bounded Bounded connected open subsets of $\mathbb{C}^n$
    - Cn conn open circular Connected circular open subsets of $\mathbb{C}^n$
    - cont Continuous functions on $\mathbb{R}^d$
    - cube dyadic Dyadic cubes
    - curves Curves
    - derivative Differentiable functions
    - forms Differential forms on $\mathbb{R}^n$
    - Fourier-Wigner Fourier-Wigner transform
    - harmonic composed conformal Harmonic functions composed with conformal maps
    - Hilbert Hilbert transform
    - hol harmonic disk-circle Fourier-Cauchy-Poisson correspondence of holomorphic and harmonic functions on the unit disk and their boundary values
    - Hol sets Holomorphic subsets of $\mathbb{C}^n$
    - hypersurf 2n reg Regular hypersurfaces in $\mathbb{R}^{2n}$
    - hypersurf or Orientable hypersurfaces in $\mathbb{R}^n$
    - KG

$$\partial_t^2 + \sum_{i=1}^{n} v_i^2 \partial_{x_i}^2 + m^2$$

    - Laplace Laplace operator on $\mathbb{R}^n$
    - Lmeas Lebesgue measurable subsets of and functions on $\mathbb{R}^n, T^n, S^n$
    - Lmeas bd of open Lebesgue measure of boundary of open sets in $\mathbb{R}^n$
    - met density Metric density of subsets of $\mathbb{R}^n$
    - Mobius n-sphere Mobius endomorphisms
    - monotone Monotone functions on $\mathbb{R}$
    - periodic int Cauchy Cauchy integral of periodic functions
    - poly int Polygons with integer vertices
    - R 2 open smooth End Open smooth maps $U \subseteq \mathbb{R}^2 \to \mathbb{C}$

- R n discrete subg Discrete subgroups of $\mathbb{R}^n$
- R n discrete subg cocpt Discrete cocompact subgroups of $\mathbb{R}^n$, flat tori
- RC ramified germs Ramified germs of smooth and holomorphic functions
- Rn open Open subsets of $\mathbb{R}^n$
- Rn open Riem Open subsets of $\mathbb{R}^n$ equipped with the flat metric
- smooth quasi-analytic Quasi-analytic smooth functions on $\mathbb{R}$
- star shaped Star-shaped subsets of $\mathbb{R}^n$
- Vec ODEs in $\mathbb{R}^n$ $\leftrightarrow$ Vector fields in $\mathbb{R}^n$
- wave

$$\partial_t^2 + \sum_{i=1}^{n} v_i^2 \partial_{x_i}^2$$

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1. multivariable calculus - Does the Poisson kernel give a unique harmonic function with given boundary data? - Mathematics Stack Exchange ↩