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Regular hypersurfaces in $\mathbb{R}^{2n}$

Definition. **Regular hypersurfaces in $\mathbb{R}^{2n}$**

A **f-regular hypersurface** in $\mathbb{R}^{2n}$ for $f \in \mathcal{C}^\infty(\mathbb{R}^{2n})$ is a subset given by

$$f^{-1}(c)$$

for some $c \in \mathbb{R}$ such that

- it is a smooth $2n-1$ dimensional submanifold
- $\mathrm{d}f \neq 0$ on $f^{-1}(c)$

closed orbits on hypersurfaces

Question

Does a compact regular H-hypersurface in $(\mathbb{R}^{2n}, \omega_{\mathrm{st}})$ possess a periodic solution of the Hamiltonian vector field X_H?

Proposition: Let $H, F \in \mathcal{C}^\infty(M)$ and

$$S = H^{-1}(c) = F^{-1}(c')$$

with

$$\mathrm{d}H, \mathrm{d}F \neq 0 \text{ on } S$$

(S is both H, F-regular hypersurface). Then the flows of X_H, X_F are

reparameterizations of one another. In particular the the periodic orbits of X_H, X_F are the same.

We have

$$\ker \mathrm{d}F = \ker \mathrm{d}H$$

and

$$\mathrm{d}F = \rho \mathrm{d}H \text{ on } S$$

for $\rho \in \mathcal{C}^\infty(S)$.

- **This makes**

$$X_F = \rho X_H$$

which implies the two vector fields have same orbits.

Attention

Somehow the hypersurface $S \subset \mathbb{R}^{2n}$ carries a prescribed collection of ω_{st}-orbits!

Current note has 0 direct children and 0 total descendants.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - hypersurf 2n reg Regular hypersurfaces in $\mathbb{R}^{2n}$

And it has 36 siblings.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - 1Hol Holomorphic functions on spaces over $\mathbb{C}$ of dimension 1
    - circle packing Circle packing on $\mathbb{R}^2$
    - circle packing to Riemann map Circle packing converges to the Riemann biholomorphism
    - Cn conn open bounded Bounded connected open subsets of $\mathbb{C}^n$
    - Cn conn open circular Connected circular open subsets of $\mathbb{C}^n$
    - cont Continuous functions on $\mathbb{R}^d$
    - cube dyadic Dyadic cubes
    - curves Curves
    - derivative Differentiable functions
    - forms Differential forms on $\mathbb{R}^n$

- Fourier-Wigner Fourier-Wigner transform
- harmonic composed conformal Harmonic functions composed with conformal maps
- Hilbert Hilbert transform
- hol harmonic disk-circle Fourier-Cauchy-Poisson correspondence of holomorphic and harmonic functions on the unit disk and their boundary values
- Hol sets Holomorphic subsets of $\mathbb{C}^n$
- hypersurf 2n reg Regular hypersurfaces in $\mathbb{R}^{2n}$
- hypersurf or Orientable hypersurfaces in $\mathbb{R}^n$
- KG

$$\partial_t^2 + \sum_{i=1}^{n} v_i^2 \partial_{x_i}^2 + m^2$$

- Laplace Laplace operator on $\mathbb{R}^n$
- Lmeas Lebesgue measurable subsets of and functions on $\mathbb{R}^n, T^n, S^n$
- Lmeas bd of open Lebesgue measure of boundary of open sets in $\mathbb{R}^n$
- met density Metric density of subsets of $\mathbb{R}^n$
- Mobius n-sphere Mobius endomorphisms
- monotone Monotone functions on $\mathbb{R}$
- periodic int Cauchy Cauchy integral of periodic functions
- poly int Polygons with integer vertices
- R 2 open smooth End Open smooth maps $U \subseteq \mathbb{R}^2 \to \mathbb{C}$
- R n discrete subg Discrete subgroups of $\mathbb{R}^n$
- R n discrete subg cocpt Discrete cocompact subgroups of $\mathbb{R}^n$, flat tori
- RC ramified germs Ramified germs of smooth and holomorphic functions
- Rn open Open subsets of $\mathbb{R}^n$
- Rn open Riem Open subsets of $\mathbb{R}^n$ equipped with the flat metric
- smooth quasi-analytic Quasi-analytic smooth functions on $\mathbb{R}$
- star shaped Star-shaped subsets of $\mathbb{R}^n$
- Vec ODEs in $\mathbb{R}^n \leftrightarrow$ Vector fields in $\mathbb{R}^n$
- wave

$$\partial_t^2 + \sum_{i=1}^{n} v_i^2 \partial_{x_i}^2$$