> **Info**
>
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> is written (completely with human hands) by Rupadarshi Ray,
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Operator norm of linear maps between $\mathbb{R}^n$

> **Definition. Operator norm of linear maps between $\mathbb{R}^n$**
>
> Let $A \in \mathrm{HomVec}_{\mathbb{R}}(\mathbb{R}^n, \mathbb{R}^m)$ we have
>
> $$\|A\| := \sup\{\|Ax\| \mid x \in \mathbb{R}^n, \|x\| \leq 1\}$$

- Any $x \in \mathbb{R}^n$ we have $x = \sum c_i e_i$, $\forall i \; |c_i| \leq \|x\| \leq 1$ then
$$\|Ax\| \leq \sum |c_i| \, \|Ae_i\| \leq \sum \|Ae_i\|$$
- $$\|Ax\| \leq \|A\| \, \|x\|$$
- $$\forall x \in \mathbb{R}^n \, (\|Ax\| \leq \lambda \|x\|) \implies \|A\| \leq \lambda$$
- $\|A+B\| \leq \sup_{\|x\| \leq 1} \|(A+B)(x)\| \leq \sup_{\|x\| \leq 1} (\|Ax\| + \|Bx\|) \leq \sup_{\|x\| \leq 1} \|Ax\| + \sup_{\|x\| \leq 1} \|Bx\| \leq \|A\|$

> $$\mathrm{HomVec}_{\mathbb{R}}(\mathbb{R}^n, \mathbb{R}^m)$$
>
> **is a normed $\mathbb{R}$-vector space with the**
>
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And because it is the preimage of the set $\mathbb{R} \setminus \{0\}$ with the map $\det : \mathrm{HomVec}_{\mathbb{R}}(\mathbb{R}^n, \mathbb{R}^m) \to \mathbb{R}$
:

$$GL_{\mathbb{R}}(\mathbb{R}^n)$$

is an open subset of $\mathrm{HomVec}_{\mathbb{R}}(\mathbb{R}^n, \mathbb{R}^m)$ **with the** operator norm **and**

$$()^{-1} : GL_{\mathbb{R}}(\mathbb{R}^n) \to GL_{\mathbb{R}}(\mathbb{R}^n)$$

is a continuous map.

- $$A \in GL(\mathbb{R}^n), \|B - A\| \left\|A^{-1}\right\| < 1 \implies B \in GL(\mathbb{R}^n)$$
  -

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- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - linear
      - operator norm Operator norm of linear maps between $\mathbb{R}^n$

And it has 1 siblings.

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