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Sequence of functions on $\mathbb{R}^d$

Definition. Sequence of functions on $\mathbb{R}^d$

uniform convergence

bounded by a summable series $\implies$ absolute and uniform convergence

Let $M_k > 0$ be a sequence of real numbers and f_k be a sequence of functions on $U \subseteq \mathbb{R}$

$$x \in U \implies |f_k(x)| \leq M_k$$

- For $n \geq m$

$$\left| \sum_{k=0}^{n} f_k(x) - \sum_{k=0}^{m} f_k(x) \right| = \left| \sum_{k=m+1}^{n} f_k(x) \right| \leq \sum_{k=m+1}^{n} M_k$$

- If

$$\sum_{k \geq 0} M_k < \infty$$

then for any $\epsilon > 0$ we have a N_ϵ such that

$$n \geq m > N_\epsilon \impliedby \sum_{m+1}^{n} M_k < \epsilon$$

then this means

$$\left| \sum_{k=0}^{n} f_k(x) - \sum_{k=0}^{m} f_k(x) \right| \leq \sum_{k=m+1}^{n} M_k < \epsilon$$

- Therefore the series (f_k) is **uniformly Cauchy** on U, thus it converges uniformly.

(Weierstrass M-test) Let for $k \in \mathbb{N}$, $f_k : U \to \mathbb{R}$ be a sequence of functions such that

$$x \in X, k \in \mathbb{N} \implies |f_k(x)| \leq M_k$$

and

$$\sum_{k \geq 0} M_k < \infty$$

Then the series

$$\sum_{k \geq 0} f_k$$

converges absolutely and uniformly on U.

Current note has 0 direct children and 0 total descendants.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - mapping
      - seqf Sequence of functions on $\mathbb{R}^d$

And it has 4 siblings.

- stamp stamp
  - Rf subobjects of and functions on $\mathbb{R}^n, T^n, S^n, \mathbb{C}^n$
    - mapping
      - balls Mapping balls
      - inverse Inverse of differentiable maps
      - mean value Mean value property of functions
      - seqf Sequence of functions on $\mathbb{R}^d$