

Info

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 is written (completely with human hands) by [Rupadarshi Ray](#),
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Asymptotics of monotonically increasing functions

$[0, \infty] \rightarrow [0, \infty]$

Let

$$f, g : [0, \infty] \rightarrow [0, \infty]$$


be functions.

Then	if	(also equivalent to)
$f \lesssim g$ or $g \gtrsim f$ or $f = O(g)$	<i>f is less than some positive scaling of g</i> $\exists c > 0 : f \leq cg$ <i>"f grows at worst g-ly"</i>	
"eventually $f \lesssim g$ "		
f is comparable to g $f \asymp g$	<i>f can be sandwiched between positive scalings of g</i> $\exists c_1, c_2 > 0 : c_1 g \leq f \leq c_2 g$	$f \lesssim g$ and $g \lesssim f$
f is negligible to g $f \ll g$ or $f = o(g)$	$\lim_{t \rightarrow \infty} \frac{f(t)}{g(t)} = 0$ $\iff \forall \epsilon > 0 \exists T > 0 : f \leq \epsilon g$ on (T, ∞) $\iff \forall \epsilon > 0, f \lesssim \epsilon g$ on (T_ϵ, ∞)	
$\rho_1 \preceq \rho_2$	<i>ρ_1 is less than some positive scaling and translate of a scaled and translated ρ_2</i> $\exists a, b > 0, c, d \geq 0 : \rho_1(r) \leq a\rho_2(cr + d)$	

Then	if	(also equivalent to)
$\rho_1 \simeq \rho_2$		$\rho_1 \preceq \rho_2$ and $\rho_2 \preceq \rho_1$
$f \overset{\rightarrow t_0}{\simeq} g$ for $t_0 \in [0, \infty]$	$\lim_{t \rightarrow t_0} \frac{f(t)}{g(t)} = 1$	

 Exercise

$$r^d \simeq r^D \implies d = D$$

 Exercise

$$e^{\alpha r} \simeq e^{\beta r} \text{ for all } \alpha, \beta > 0$$

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 - [asympt](#) Asymptotics of monotonically increasing functions $[0, \infty] \rightarrow [0, \infty]$

And it has 1 siblings.

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