Info

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Metric simplicial complexes

A *simplex* is a metric space

$$(S, d_S)$$

which comes with a collection of subsets

$$\mathrm{faces}(S) \subset 2^S$$

called *faces*.

Definition. Gluing of faces

Let

$$(S_i, d_{S_i}, \mathrm{faces}(S_i))$$

be a C-simplices for $i = 1, 2$. A **gluing** of S_1, S_2 is an isometry

$$\phi : (F_1, d_S) \xrightarrow{\text{isometry}} (F_2, d_S)$$

for some $F_1 \in \mathrm{faces}(S_1), F_2 \in \mathrm{faces}(S_2)$.
The set of all gluings is denoted as

$$\mathrm{gluings}(S_1, S_2)$$

Definition. Simplicial complex

Let $\mathcal{X}$ be a family of C-simplices and $\mathcal{G}$ be a family of *gluings* of pairs in $\mathcal{X}$.

Construct the space

$$X := \frac{\coprod \mathcal{X}}{\mathcal{G}}$$

with projection

$$\pi : \coprod \mathcal{X} \to X$$

and metric

$$d_X := \pi^* \overline{\coprod_{S \in X} d_S}$$

The metric space (X, d_X) is a **simplicial complex** if

- no simplex is glued to itself $\iff$

$$\pi|_S : S \to X$$

  is injective for all $S \in \mathcal{X}$.
- every simplex is glued to some other simplex

The set of $\mathcal{G}$-classes of the faces of $S \in \mathcal{X}$ is denoted

$$\mathrm{shapes}(X) := \frac{\bigcup \mathrm{faces}(S)}{\mathcal{G}}$$

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Current note has 1 direct children and 1 total descendants.

- structures based on sets
  - Metric spaces
    - Metric simplicial complexes
      - Metric cube complexes

And it has 11 siblings.

- structures based on sets
  - Metric spaces
    - Set of all metrics on a set
    - CAT(0)
    - Space of all compact metric spaces up to isometry
    - Y^X
    - Gromov hyperbolic spaces
    - Metric space isometries
    - Constructing more metrics
    - Metric spaces
    - Metric simplicial complexes
    - Metric subspaces
    - Uniformly discrete spaces of bounded geometry