> ⓘ Info
>
> This note, that was found here, as a part of a collection
> is written (completely with human hands) by Rupadarshi Ray,
> was created on February 25, 2026 12:09:51 AM,
> was last modified on August 28, 2026 5:00:59 PM, and
> was exported on September 7, 2026 3:09:46 PM.

Decomposition of hyperbolic surfaces

		area	geodesic boundary
$P(0)$		finite	$\emptyset$
$P(1)(l_1)$		finite	one compact component of length l_1
$P(2)(l_1, l_2)$		finite	two compact components of lengths l_1, l_2
$P(3)(l_1, l_2, l_3)$		finite	three compact components of length l_1, l_2, l_3
half-A_m		∞	one compact component
half-$\mathbb{R}\boldsymbol{H}^2$		∞	one non-compact component which is a geodesic line

> **Definition.** A **hyperbolic surface with geodesic boundary** is a 2-manifold with boundary such that each interior point has a onb isometric to an open subset of $\mathbb{R}\boldsymbol{H}^2$ and each boundary point has a onb isometric to a onb of a point on the boundary geodesic in half-$\mathbb{R}\boldsymbol{H}^2$.

number of boundary components	surfaces with boundary whose interior is homeomorphic to $T^2_{0,3}$
0	$T^2_{0,3}$
1	$T^2_{0,2,1}$
2	$T^2_{0,1,2}$
3	$T^2_{0,0,3}$ (compact)

case	if	then	decomposition
compact connected hyperbolic surface with geodesic boundary	such that simple closed geodesics are boundary components	$X \cong_{\text{Top}} T^2_{0,0,3}$	$P(3)$
non-compact connected hyperbolic surface **without boundary** such that every simple closed curve in X homotopic to a point or to ∂X or every simple closed curve in X bounds a parabolic cylinder	simply connected	$X \cong_{\text{Riem}} \mathbb{R}\boldsymbol{H}^2$	half-$\mathbb{R}\boldsymbol{H}^2$+half-$\mathbb{R}\boldsymbol{H}^2$
	$\pi_1(X, x_0) \cong \mathbb{Z}$	$X \cong_{\text{Riem}} \langle z \mapsto z+1 \rangle \backslash^{\mathbb{R}}$ parabolic cylinder	half-$\mathbb{R}\boldsymbol{H}^2$+$P(0)$
	$\pi \ncong \{1\}$ or $\mathbb{Z}$	$X \cong_{\text{Top}} T^2_{0,3}$ with three cusps $X \cong_{\text{Riem}} \Gamma(2) \backslash^{\mathbb{R}\boldsymbol{H}^2}$	$P(0)$
non-compact connected hyperbolic surface (not classified above) **without boundary**	$\pi_1(X, x_0) \cong \mathbb{Z}$	annulus $X \cong_{\text{Riem}} \langle z \mapsto \lambda z \rangle \backslash^{\mathbb{R}\boldsymbol{H}}$	half-A_m+half-A_m
non-compact connected hyperbolic surface with **compact** (non-empty) geodesic boundary such that every simple closed curve in X homotopic to a point or ∂X or to every simple closed curve in X bounds a parabolic cylinder	**one boundary component** $\partial X = A$ and every simple closed curve in X homotopic to a point or to $\partial X = A$	half-annulus	half-A_m
	one boundary component $\partial X = A$ there is a curve B which is not homotopic to point or A	$X \cong_{\text{Top}} T^2_{0,2,1}$ with two cusps	$P(1)$

case	if	then	decomposition
	at least two boundary components A, B	$X \cong_{\text{Top}} T^2_{0,1,2}$ with one cusp	$P(2)$
		half-$\mathbb{R}\boldsymbol{H}^2$	half-$\mathbb{R}\boldsymbol{H}^2$

- Suppose X has two distinct boundary components

$$A \sqcup B \subseteq \partial X$$

  - As A, B are compact 1-manifolds they are homeomorphic to S^1.
  - As X is connected there is a arc $\gamma([0,1]) = C$ joining A to B.
  - Let U be a small onb of $A \cup B \cup C$ with smooth boundary and let $D = \partial U$.
  - ...

For non-compact, connected hyperbolic surfaces

- Let X has empty boundary and $\pi \ncong \{1\}$ or $\mathbb{Z}$.
  - ... such that every simple closed curve in X bounds a punctured disk.
  - Let A bounds U_1 which is isometric to parabolic cylinder
  - $B \subset X \setminus U_1$ bounds U_2 isometric to parabolic cylinder
  -

multicurve decomposition

Definition. Multicurve on surfaces with boundary

A family $\mathfrak{Y}$ of simple closed curves on surface S is called a **multicurve** if

Let X be a non-simply connected, connected hyperbolic surface (without boundary). Then there exists a multicurve $\mathfrak{Y}$ on X such that each component of $X \setminus \overline{\mathfrak{Y}(X)}$ is isometric to one of

		area	geodesic boundary
$P(0)$		finite	$\emptyset$
$P(1)(l_1)$		finite	one compact component of length l_1
$P(2)(l_1, l_2)$		finite	two compact components of lengths l_1, l_2
$P(3)(l_1, l_2, l_3)$		finite	three compact components of length l_1, l_2, l_3
half-A_m		∞	one compact component
half-$\mathbb{R}\boldsymbol{H}^2$		∞	one non-compact component which is a geodesic line

---

Current note has 0 direct children and 0 total descendants.

- stem
  - Objects stemming from $PSL(2)(\mathbb{R})$
    - Fuchsian groups $\Gamma <_d PSL(2)(\mathbb{R}) \leftrightarrow$ oriented hyperbolic 2-manifold/orbifolds $\to$ Riemann surfaces with branching signature
      - Decomposition of hyperbolic surfaces

And it has 17 siblings.

- stem
  - Objects stemming from $PSL(2)(\mathbb{R})$
    - Fuchsian groups $\Gamma <_d PSL(2)(\mathbb{R}) \leftrightarrow$ oriented hyperbolic 2-manifold/orbifolds $\to$ Riemann surfaces with branching signature
      - Artihmetic Fuchsian groups and Artihmetic hyperbolic 2-orbifolds
      - Cocompact Fuchsian groups
      - Central simple algebra
      - Elementary Fuchsian groups
      - Finitely generated Fuchsian groups $\leftrightarrow$ geometrically finite Fuchsian groups
      - Fuchsian groups of finite coarea
      - Geodesic flow of Fuchsian groups and hyperbolic surfaces
      - Decomposition of hyperbolic surfaces
      - Infinitely generated Fuchsian groups
      - Limit set of Fuchsian groups
      - Marked hyperbolic surfaces
      - subst.R PSL 2 discrete subg.over Q-bar
      - Fuchsian Schottky groups
      - Spectrum of hyperbolic surfaces
      - Torsion-free Fuchsian groups
      - Cocompact torsion-free Fuchsian groups
      - Fuchsian triangle groups