ⓘ Info

This note, that was found here, as a part of a collection
is written (completely with human hands) by Rupadarshi Ray,
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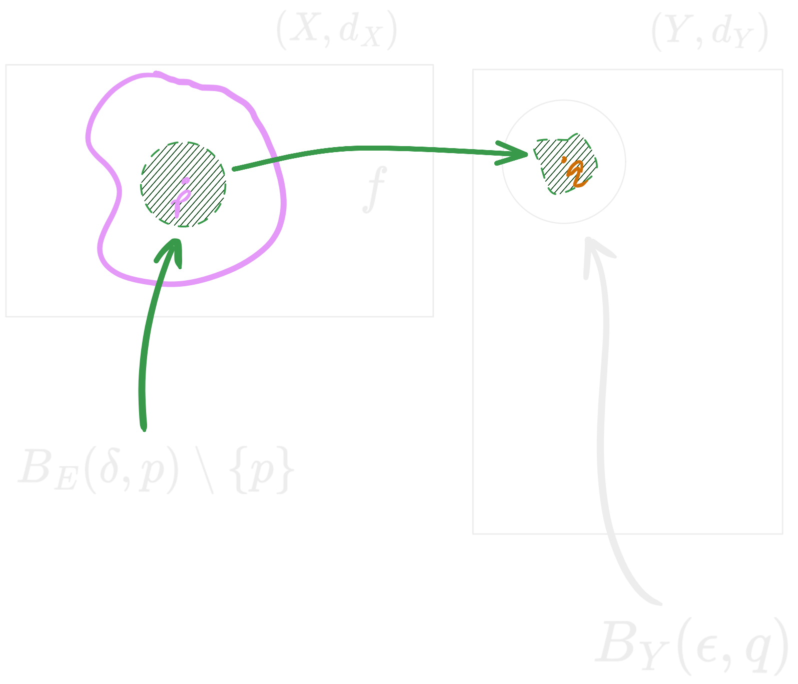
Limit of a function between metric spaces

Definition. Limit (at a point) of a function between metric spaces

Let $f : E \subseteq X \to Y$ is function between metric spaces and $p \in E$.
If $\exists q \in Y$ such that $\forall \epsilon, \exists \delta$ satisfying

$$f(\delta\text{–}B_E(p) \setminus \{p\}) \subseteq \epsilon\text{–}B_Y(q)$$



by the function f, then the limit of the function f at p is q,

$$\lim_{x \in E \to p} f(x) = q$$

A generalised definition, for the limit of a function in the subset $A \subseteq E$ is

Definition. Limit (in a subset) of a function between metric spaces

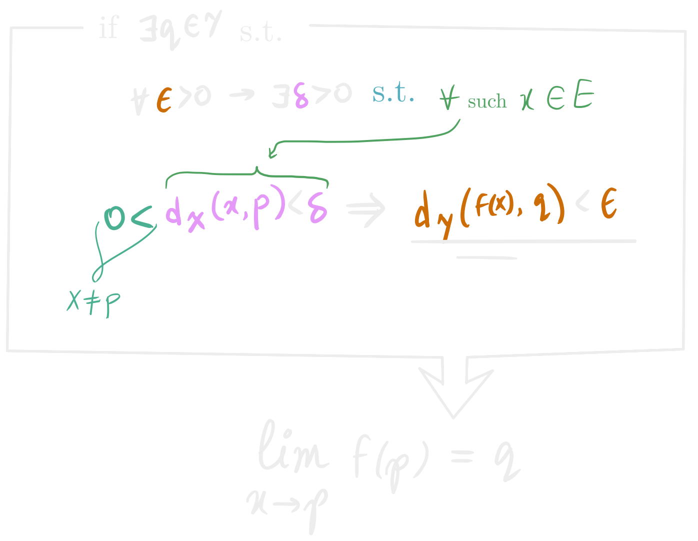
Let $f : E \subseteq X \to Y$ is function between metric spaces and $p \in E$.
If $\exists q \in Y$ such that $\forall \epsilon, \exists \delta$ satisfying

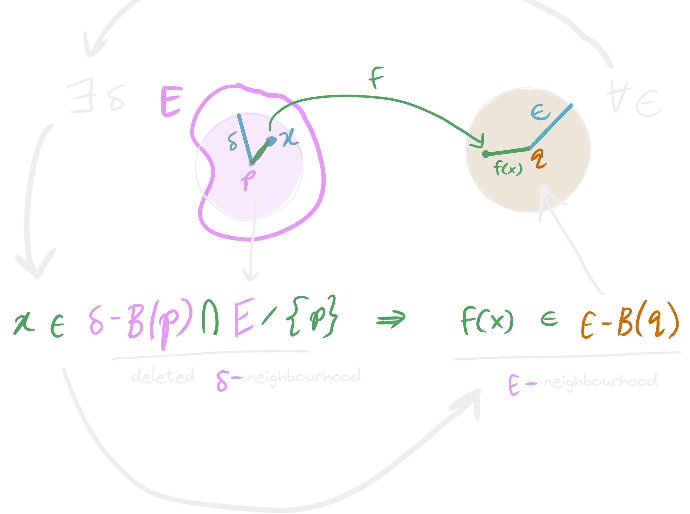
$$(\delta\text{–}B_E(p) \setminus \{p\}) \cap A \subseteq E \mapsto \epsilon\text{–}B_Y(q)$$

by the function f, then the limit of the function f in the subset $A \subseteq E$ is q,

$$\lim_{x \in A \subseteq E \to p} f(x) = q$$

Definition. Limit of a function at a point

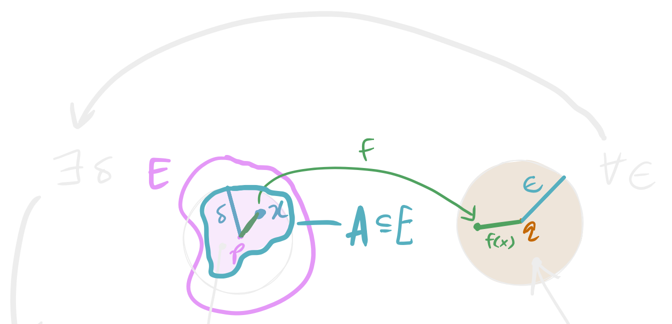




if ∀ε>0 ⟶ ∃δ>0 st.
For all x ∈ deleted δ−neighbourhood(p)
f(x) ∈ ε−neighbourhood(q)
then $\lim_{x \to p} f(x) = q$

Definition. Limit of a function in a subset

Slightly general def^n



$$\underbrace{x \in \delta\text{-}B(p) \cap A}_{\text{deleted } \delta\text{-neighbourhood in } A} \Rightarrow f(x) \in \underbrace{\epsilon\text{-}B(q)}_{\epsilon\text{-neighbourhood}}$$

$$\Downarrow$$

$$\lim_{x \in A \to p} f(x) = q$$

Current note has 0 direct children and 0 total descendants.

- structures based on sets
  - Metric spaces
    - Metric spaces
      - Function between metric spaces
        - Limit of a function between metric spaces

And it has 6 siblings.

- structures based on sets
  - Metric spaces
    - Metric spaces
      - Function between metric spaces
        - Continuous functions between metric spaces
        - Uniformly continuous functions on a metric space
        - Contraction map in a metric space
        - Topological entropy of maps between metric spaces
        - Limit of a function between metric spaces
        - Sensitive dependence on initial conditions of dynamics of a map between metric space