

This note, that [was found here](#), as a part of [a collection](#) is written (completely with human hands) by [Rupadarshi Ray](#), was created on May 27, 2026 10:38:26 PM, was last modified on September 7, 2026 1:10:46 PM, and was exported on September 7, 2026 3:09:56 PM.

Hilbert transform

Let $\psi \in \mathcal{S}(\mathbb{R}^d)$ and consider the integral



$$\int_{[-\epsilon,\epsilon]} \frac{\psi(x)}{x} \mathrm{d}x$$

The function $\frac{1}{x}$ is not integrable on $[-\epsilon,\epsilon]$. As ψ is bounded on $[-\epsilon,\epsilon]$,

$$\frac{M_0}{|x|} \leq \frac{|\psi(x)|}{|x|}$$

so $\frac{\psi(x)}{x}$ is also not integrable on $[-\epsilon,\epsilon]$.

However, as $\frac{1}{x}$ is odd, we have

$$\int_{\epsilon \leq |x| \leq 1} \frac{\psi(x)}{x} \mathrm{d}x = \int_{\epsilon \leq |x| \leq 1} \frac{\psi(x) - \psi(0)}{x} \mathrm{d}x$$

and by [mean value theorem](#) we have

$$\left| \frac{\psi(x) - \psi(0)}{x} \right| \leq \|\psi'\|_\infty$$

Thus,

$$0 \leq \left| \int_{\epsilon \leq |x| \leq 1} \frac{\psi(x)}{x} \mathrm{d}x \right| \leq \|\psi'\|_\infty (2\epsilon)$$

so the limit exists

$$\lim_{\epsilon \rightarrow 0} \int_{\epsilon \leq |x| \leq 1} \frac{\psi(x)}{x} \mathrm{d}x$$

 Definition.

$$\begin{aligned} \mathcal{S}(\mathbb{R}^d) &\rightarrow \mathbb{C} \\ \psi &\mapsto \lim_{\epsilon \rightarrow 0} \int_{\epsilon \leq |x| \leq 1} \frac{\psi(x)}{x} + \int_{|x| \geq 1} \frac{\psi(x)}{x} \end{aligned}$$

Current note has 0 direct children and 0 total descendants.

- [stem](#)
 - [subobjects of and functions on \$\mathbb{R}^n, S^n, \mathbb{C}^n\$ and its quotients \$\Gamma \backslash \mathbb{R}^n, \Gamma \backslash \mathbb{C}^n\$](#)
 - [Hilbert transform](#)

And it has 39 siblings.

- [stem](#)
 - [subobjects of and functions on \$\mathbb{R}^n, S^n, \mathbb{C}^n\$ and its quotients \$\Gamma \backslash \mathbb{R}^n, \Gamma \backslash \mathbb{C}^n\$](#)
 - [Holomorphic functions on spaces over \$\mathbb{C}\$ of dimension 1](#)
 - [Open subsets of \$\mathbb{R}^n\$ with \$\mathcal{C}^2\$ boundary](#)
 - [Complex ODEs in \$\mathbb{C}\$](#)
 - [Circle packing on \$\mathbb{R}^2\$](#)
 - [Circle packing converges to the Riemann biholomorphism](#)
 - [Bounded connected open subsets of \$\mathbb{C}^n\$](#)
 - [Connected circular open subsets of \$\mathbb{C}^n\$](#)
 - [Continuous functions on \$\mathbb{R}^d\$](#)
 - [Dyadic cubes](#)
 - [Curves](#)
 - [Differentiable functions](#)
 - [Differential forms on \$\mathbb{R}^n\$](#)
 - [Fourier-Wigner transform](#)
 - [Harmonic functions composed with conformal maps](#)
 - [Hilbert transform](#)
 - [Fourier-Cauchy-Poisson correspondence of holomorphic and harmonic functions on the unit disk and their boundary values](#)
 - [Holomorphic subsets of \$\mathbb{C}^n\$](#)
 - [Regular hypersurfaces in \$\mathbb{R}^{2n}\$](#)
 - [Orientable hypersurfaces in \$\mathbb{R}^n\$](#)
 - [Laplace operator on \$\mathbb{R}^n\$](#)
 - [l[∞]\(\$\mathbb{R}\$ \)](#)
 - [Lebesgue measurable subsets of and functions on \$\mathbb{R}^n, T^n, S^n\$](#)
 - [Lebesgue measure of boundary of open sets in \$\mathbb{R}^n\$](#)
 - [Local \$\mathbb{R}\$ -algebras](#)
 - [l^p\(\$\mathbb{R}\$ \)](#)
 - [Metric density of subsets of \$\mathbb{R}^n\$](#)
 - [Monotone functions on \$\mathbb{R}\$](#)
 - [Cauchy integral of periodic functions](#)
 - [Polygons with integer vertices](#)
 - [Open smooth maps \$U \subseteq \mathbb{R}^2 \rightarrow \mathbb{C}\$](#)

- Discrete subgroups of $\mathbb{R}^n$
- Discrete cocompact subgroups of $\mathbb{R}^n$, flat tori
- Ramified germs of smooth and holomorphic functions
- Open subsets of $\mathbb{R}^n$
- Open subsets of $\mathbb{R}^n$ equipped with the flat metric
- Closed subsets of $\mathbb{R}^n$ equipped with smooth functions
- Quasi-analytic smooth functions on $\mathbb{R}$
- Star-shaped subsets of $\mathbb{R}^n$
- ODEs in $\mathbb{R}^n \leftrightarrow$ Vector fields in $\mathbb{R}^n$