Info

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Connection on the tangent bundle of a smooth manifold

Motivation: Any derivative of vector fields on a local chart is not chart independent.

Definition. Connection on the tangent bundle of a smooth manifold

Let M be a smooth manifold. A **connection** on M is any $\mathbb{R}$-bilinear map,

$$\begin{aligned} \nabla : \boldsymbol{\Gamma} TM \times \boldsymbol{\Gamma} TM &\to \boldsymbol{\Gamma} TM \\ (X, Y) &\mapsto \nabla_X Y \end{aligned}$$

such that

$$\begin{aligned} \nabla_{fX} Y &= f(\nabla_X Y) \\ \nabla_X (fY) &= (Xf)Y + f(\nabla_X Y), \end{aligned}$$

for all $X, Y \in \boldsymbol{\Gamma} TM$ and all $f \in C^\infty(M)$. The vector field, $\nabla_X Y$, is called the **covariant derivative** of Y with respect to X.

what can we differentiate using a connection?

tensor fields	
vector fields along curves	

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