Info

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Measure homology on a hyperbolic manifold

[1]

For $k \in \mathbb{N}$ let

$$\mathcal{C}^\infty(\Delta^k, M)$$

be equipped with the following topology

- for $k = 0$, let it be the compact-open topology
- for $k > 1$, let it be generated from the basis

$$N(\sigma, r)$$

Definition. Definition

$$\mathrm{meas}_k(M) := \mathrm{Meas}_c(\mathfrak{B}_{\mathcal{C}^\infty(\Delta^k, M)}, \mathbb{R})$$

For each $i \in \{0, \ldots, k\}$ we have the **i-th face map**

$$\not{f}_i : \Delta^{k-1} \hookrightarrow \Delta^k$$

which induces the continuous map

$$\begin{aligned} \not{f}_i^* : \mathcal{C}^\infty(\Delta^k, M) &\to \mathcal{C}^\infty(\Delta^{k-1}, M) \\ \sigma &\mapsto \not{f}_i^*(\sigma) := \sigma \circ \not{f}_i \end{aligned}$$

that in turn induces a linear map

$$(\not{f}_i^*)_* : \mathrm{meas}_k(M) \to \mathrm{meas}_{k-1}(M)$$

We define the boundary map

$$\partial_k := \sum_{i=1}^{k} (-1)^i (\not{f}_i^*)_*$$

This constructs a chain complex

$$\cdots \to \mathrm{meas}_k(M) \xrightarrow{\partial_k} \mathrm{meas}_{k-1}(M) \to \ldots$$

whose *homology* is called the **measure homology** of M.

> **Proposition:** is a chain complex

Current note has 0 direct children and 0 total descendants.

- stem
  - O+ R1 n
    - Discrete subgroups of $O^+(1, n) \leftrightarrow$ hyperbolic n-manifolds/orbifolds
      - Measure homology on a hyperbolic manifold

And it has 11 siblings.

- stem
  - O+ R1 n
    - Discrete subgroups of $O^+(1, n) \leftrightarrow$ hyperbolic n-manifolds/orbifolds
      - Convex cocompact discrete subgroups
      - Combination of discrete subgroups of $O^+(1, n)(\mathbb{R})$
      - Exponential sum of a discrete subgroup of $\mathrm{Isom}(\mathbb{R}\boldsymbol{H}^n)$
      - $\mathrm{RepDF}(F_g, O^+(1, n)(\mathbb{R}))$
      - Measure homology on a hyperbolic manifold
      - Discrete subgroups with limit set a discontinuum
      - Limit sets of discrete subgroups
      - Discrete subgroups with limit set a $k-1$-sphere
      - Counting orbits of discrete subgroups of $O^+(1, n)(\mathbb{R})$ in $\mathbb{R}\boldsymbol{H}^n$
      - Rigidity of $\mathbb{R}$-hyperbolic manifolds
      - $\mathrm{RepDF}(\pi_1(T_g^2, \bullet), O^+(1, n)(\mathbb{R}))$

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1. John G. Ratcliffe - Foundations of Hyperbolic Manifolds (2006), p.575 ↩