ⓘ Info

This note, that was found here, as a part of a collection
is written (completely with human hands) by Rupadarshi Ray,
was created on May 21, 2025 4:05:54 PM,
was last modified on September 7, 2026 1:11:06 PM, and
was exported on September 7, 2026 3:09:57 PM.

Fourier transform on $\mathbb{R}^n$

Definition. Fourier transform on $L^1(\mathbb{R}^n)$

The **Fourier transform** on $L^1(\mathbb{R}^n)$ is

$$\widehat{} : L^1(\mathbb{R}^n) \to \mathcal{C}_0(\mathbb{R}^n)$$
$$f \mapsto \hat{f}(\xi) := \int_{x \in \mathbb{R}^n} f(x) \exp(-2\pi i \langle x, \xi \rangle)$$

$L^1(\mathbb{R}^n)$	$\leftrightarrow$	$\widehat{L^1}(\mathbb{R}^n) \leq \mathcal{C}_0(\mathbb{R}^n)$
$L^2(\mathbb{R}^n)$	$\leftrightarrow$	$L^2(\mathbb{R}^n)$
$L^1 \cap L^2$	$\hookrightarrow$	$\mathcal{C}_0 \cap L^2$
$\mathcal{S}$	$\leftrightarrow$	$\mathcal{S}$
$\mathcal{S}^\star_c$	$\hookrightarrow$	
L^∞	$\leftrightarrow$	
$\mathcal{S}^\star$	$\leftrightarrow$	$\mathcal{S}^\star$

$L^2(-A, A)$	[1] $\leftrightarrow$	$L^2(\mathbb{R}) \cap \Big\{F \in \mathscr{O}(\mathbb{C}) \Big\vert \lvert F(z) \rvert \lesssim$
$\mathcal{C}(\mathbb{R})_{\lesssim \frac{1}{1+x^2}} \cap \mathscr{O}(\mathbb{C})_{\lesssim e^{2\pi M \lvert z \rvert}}$	$\leftrightarrow$	$\widehat{\mathcal{C}(\mathbb{R})_{\lesssim \frac{1}{1+x^2}}} \cap \mathcal{C}[-M, M]$

Current note has 0 direct children and 0 total descendants.

- stem
  - subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $_\Gamma\backslash^{\mathbb{R}^n}, {}_\Gamma\backslash^{\mathbb{C}^n}$
    - Fourier
      - Fourier transform on $\mathbb{R}^n$

And it has 10 siblings.

- stem
  - subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $_\Gamma\backslash^{\mathbb{R}^n}, {}_\Gamma\backslash^{\mathbb{C}^n}$
    - Fourier
      - Fourier transform on $L^2(-A, A) \leq L^2(\mathbb{R})$
      - $\widehat{} : L^2(0, \infty) \cong_{\text{Hilb}} \mathscr{O}^2(H^2_{\text{U}})$
      - Fourier transform on $\mathbb{R}^n$
      - Fourier transform on S^1, Fourier series on $[0, 1]$
      - Functions on S^1 with absolutely converging Fourier series, $\check{l}^1(S^1)$
      - Fourier transform of distributions on S^1
      - $\widehat{} : L^1[0, 1] \to \mathcal{C}_0(\mathbb{Z}, \mathbb{C})$
      - $\widehat{} : L^2[0, 1] \cong_{\text{Hilb}} l^2(\mathbb{Z}, \mathbb{C})$
      - Fourier transform of measurable subsets
      - Unsharpness principles

---

1. **(Paley-Wiener)**

$$\hat{} : L^2(-A, A) \cong L^2(\mathbb{R}) \cap \{F \in \mathscr{O}(\mathbb{C}) \vert \lvert F(\bullet) \rvert \lesssim \exp(A \lvert \bullet \rvert) \text{ on } \mathbb{C}\}$$

↩