Info

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Angle form on $\mathbb{R}^2 \setminus \{0\}$

#wiki/form

Definition. Angle form on $\mathbb{R}^2 \setminus \{0\}$

The angle form on $\mathbb{R}^2 \setminus \{0\}$ is defined by

$$\frac{-y\mathrm{d}x + x\mathrm{d}y}{x^2+y^2}$$

using the coordinates (x, y).

$$\begin{aligned} \mathrm{d}\left(\tan^{-1}\left(\frac{y}{x}\right)\right) &= \frac{1}{1+\left(\frac{y}{x}\right)^2}\frac{-y\mathrm{d}x}{x^2} + \frac{1}{1+\left(\frac{y}{x}\right)^2}\frac{\mathrm{d}y}{x} \\ &= \frac{-y\mathrm{d}x + x\mathrm{d}y}{x^2+y^2} \end{aligned}$$

the pullback of inexact closed forms on punchered plane

$$H^1(\Omega^\bullet(\mathbb{R}^2 \setminus \{0\})) = \mathbb{R}\left[\frac{-y\mathrm{d}x + x\mathrm{d}y}{x^2+y^2}\right]$$

$$\pi^* \frac{-y\mathrm{d}x + x\mathrm{d}y}{x^2+y^2} = \mathrm{d}\theta$$

the symplectic vector field on $\mathbb{R}^2 \setminus \{0\}$ which is *not* Hamiltonian

We want the vector field $X = X_1\partial_1 + X_2\partial_2$ such that

$$\begin{aligned} \mathrm{d}x \wedge \mathrm{d}y(X_1\partial_1 + X_2\partial_2, -) &= \tfrac{-y\mathrm{d}x + x\mathrm{d}y}{x^2+y^2}(-) \\ X_1\mathrm{d}x \wedge \mathrm{d}y(\partial_1, -) + X_2\mathrm{d}x \wedge \mathrm{d}y(\partial_2, -) &= \tfrac{-y\mathrm{d}x + x\mathrm{d}y}{x^2+y^2}(-) \\ X_1 &= \tfrac{x}{x^2+y^2} \\ -X_2 &= -\tfrac{y}{x^2+y^2} \end{aligned}$$

Thus we have

$$\frac{1}{r}\frac{\partial}{\partial r} = \frac{1}{x^2+y^2}(x\partial_1 + y\partial_2)$$

Current note has 0 direct children and 0 total descendants.

- stretchy
  - $\mathbb{R}^2 \setminus \{0\}$
    - Angle form on $\mathbb{R}^2 \setminus \{0\}$

And it has 2 siblings.

- stretchy
  - $\mathbb{R}^2 \setminus \{0\}$
    - Angle form on $\mathbb{R}^2 \setminus \{0\}$
    - de Rham cohomology of the punctured plane