

This note, that [was found here](#), as a part of [a collection](#) is written (completely with human hands) by [Rupadarshi Ray](#), was created on January 13, 2025 6:19:20 PM, was last modified on September 7, 2026 1:11:13 PM, and was exported on September 7, 2026 3:09:59 PM.

A holomorphic map branched over $\mathbb{Q}$

Let $\Omega := \mathbb{C} \setminus \{0\}$ and $B := \{\frac{1}{n} \mid n \in \mathbb{Z}_{>0}\}$ so set of limit points of B is $\{0\}$ which is outside Ω . Now consider an enumeration (q_n) of $\mathbb{Q}$ and the polynomials

$$P_{\frac{1}{n}}(z) := q_n + \frac{1}{2}z^2$$

By

Let Ω be a open subset of $\mathbb{C}$ and $B \subseteq \Omega$ with no limit points in Ω . For each $b \in B$ let $P_b \in \mathbb{C}[z]$ be a polynomial. Then there is a holomorphic function

$$f : \Omega \rightarrow \mathbb{C}$$

such that the *truncated (first few terms of the) Taylor series* of f at b matches the polynomial P_b , that is

$$\sum_{k \leq \deg P_b} \frac{f^{(k)}(b)(z-b)^k}{k!} = P_b(z-b)$$

for each $b \in B$.

there exists a holomorphic function

$$f : \mathbb{C} \setminus \{0\} \rightarrow \mathbb{C}$$

such that

$$f\left(\frac{1}{n}\right) = q_n$$

the Taylor series of f at $\frac{1}{n}$ is $P_{\frac{1}{n}}\left(z - \frac{1}{n}\right)$.

So in particular

•
$$f'\left(\frac{1}{n}\right) = 0, f''\left(\frac{1}{n}\right) = 1$$

Thus we observe every point of

$$\mathbb{Q} \subseteq f(\mathbb{C} \setminus \{0\})$$

is a **branch point** of f ^[1]

[2]

Current note has 0 direct children and 0 total descendants.

- [stem](#)
 - [subobjects of and functions on \$\mathbb{R}^n, S^n, \mathbb{C}^n\$ and its quotients \$\Gamma \backslash \mathbb{R}^n, \Gamma \backslash \mathbb{C}^n\$](#)
 - [Holomorphic functions on spaces over \$\mathbb{C}\$ of dimension 1](#)
 - [mapping](#)
 - [A holomorphic map branched over \$\mathbb{Q}\$](#)

And it has 13 siblings.

- [stem](#)
 - [subobjects of and functions on \$\mathbb{R}^n, S^n, \mathbb{C}^n\$ and its quotients \$\Gamma \backslash \mathbb{R}^n, \Gamma \backslash \mathbb{C}^n\$](#)
 - [Holomorphic functions on spaces over \$\mathbb{C}\$ of dimension 1](#)
 - [mapping](#)
 - [Bounding derivative of holomorphic mappings](#)
 - [Bounding holomorphic functions linearly](#)
 - [Branch points and ramification index of holomorphic functions](#)
 - [Disk in holomorphic image of disk](#)
 - [Size of holomorphic image of disks](#)
 - [Maximum and minimum of holomorphic functions](#)
 - [Fibers of holomorphic mappings](#)
 - [Non-constant holomorphic functions locally look like \$w^k\$ and are open mappings](#)
 - [Biholomorphic mapping onto disk](#)
 - [Conformal radius of pointed simply connected open subsets of \$\mathbb{C}\$](#)
 - [Holomorphic functions on punched disk](#)
 - [Holomorphic function whose singularities are not isolated](#)
 - [A holomorphic map branched over \$\mathbb{Q}\$](#)

1. **Definition.** Branch points and ramification index of holomorphic functions

Let $\Omega \subseteq \mathbb{C}$ be a connected open set and

$$f : \Omega \rightarrow \mathbb{C}$$

be a non-constant holomorphic function. Then zeros of f' , that is, $(f')^{-1}(0)$ are the set of *critical points* of f and in particular this set has no limit points in Ω as f is holomorphic.

The **ramification index** of f is equivalently

- Around a small disk $B(z_0, r)$ of such a zero point $z_0 \in (f')^{-1}(0)$ which does not contain any other zeros, we have the winding number of the curve $f(\gamma)$
- order of the zero z_0 of f'

If the ramification index at z_0 is greater than 1 then z_0 is called a **ramification point** of f , the *critical value* $f(z_0)$ is called an **(algebraic) branch point** of f .

↩

2. <https://mathoverflow.net/a/60681> ↩