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Exponential of vector fields

Definition. Flow and exponential of a vector field on a smooth manifold

Flow of a vector field $X \in \text{Vec}(M)$ is the *flow* on M , that is, the smooth map

$$(t, -) \mapsto (\exp tX)(-)$$

that satisfies

$$\frac{d}{dt}\exp tX\Big|_{t=0}(p) = X_p, \quad \exp 0 = \text{Id}_M$$

Then X is the **generator of the flow** and

$$\exp X : M \rightarrow M$$

is the **exponential of the vector field** X .

Equivalently,

$$\frac{d}{dt}\exp tX = X_{\exp tX}$$

commutators

- BCH
- $\exp \alpha[X, Y] = \exp[\sqrt{\alpha}X, \sqrt{\alpha}Y] = [\exp \sqrt{\alpha}X, \exp \sqrt{\alpha}Y]$

complete vector fields

 A **compactly** supported smooth vector field on a smooth manifold is **complete**.

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