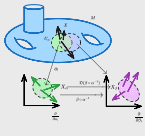


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Comparison of constructions with tangent vectors

	$X_p \in T_pM$	T_pM	TM	$X \in \Gamma TM$	ΓTM	$\text{Vec}(-)$
coordinate-free definition	$(-\circ \gamma)'(0)$	Definition Given a smooth manifold M, and any $p \in M$ we define the tangent space T_pM at the point p as $(T_pM, +, \cdot)$ containing the operators $(-\circ \gamma)'(0)$ for any smooth curve $\gamma : (-\epsilon, \epsilon) \rightarrow M$ with $\gamma(0) = p$.		Definition A smooth /continuous/rough tangent vector field on a smooth manifold M (with/out boundary) is a <i>section</i> of the projection $TM \rightarrow M$, that is, a <i>smooth /continuous/arbitrary</i> map $X : M \rightarrow TM$ satisfying $\pi_{TM} \circ X = \text{id}_M$.		

	$X_p \in T_pM$	T_pM	TM	$X \in \Gamma TM$	ΓTM	$\text{Vec}(-)$
in a chart		$T_pM \rightarrow \mathbb{R}^n$		<i>Rough</i> tangent vector field X on a smooth manifold M is a function $X : M \rightarrow \prod_{p \in M} T_pM$ $p \mapsto X_p$ It is <i>continuous</i> <i>/smooth</i> if for all charts $(U_\alpha, \underbrace{\alpha}_{U_\alpha \rightarrow \mathbb{R}^{\dim M}})$, $p \in U$, X_p written in the coordinate basis for T_pM given by α $X_p = \sum_{i=1}^{\dim M} X^i \frac{\partial}{\partial x^i}$ makes the map $X_\alpha : U_\alpha \rightarrow \prod_{p \in U_\alpha} T_p\mathbb{R}^n$ $p \mapsto X_p$ is a <i>continuous</i> <i>/smooth</i> map, which means $X_\alpha \circ \alpha^{-1}$ must be <i>continuous</i> <i>/smooth</i> map on $\alpha(U)$.		
equivalence classes of curves	$[\gamma'(0)]$					
velocities of curves	$\gamma'(t) \in T_{\gamma(t)}$	curves upto same velocity is bijective to TM	γ' is a smooth curve on TM			
curves associated with a particular chart velocity						
local basis/coord inates	$X_p = \sum_i X^i \frac{\partial}{\partial x^i} _p$	$\left\{ \frac{\partial}{\partial x^i} _p \right\}$ is a basis	elements look like $(p, \sum_i X^i \frac{\partial}{\partial x^i} _p)$	$X = \sum_i X^i \frac{\partial}{\partial x^i}$		
change of basis				Given the collection of <i>rough/con</i> <i>tinuous/s</i> <i>mooth</i> maps (U_α, α) is a atlas is a vector field on M $\iff$ $(U_\alpha, \alpha), (U_\beta, \beta)$ 		

	$X_p \in T_p M$	$T_p M$	TM	$X \in \Gamma TM$	ΓTM	$\text{Vec}(-)$
action on smooth maps, equivalent to d	$X_p(f) = \text{d}_p f$	action of vectors on smooth maps is a derivation $X_p : \mathcal{C}^\infty(M)$		$Xf = \text{d}f(X)$	action of vector fields on $\mathcal{C}^\infty(M)$ is a derivation $X : \mathcal{C}^\infty(M) \rightarrow \mathcal{C}^\infty(M)$	
derivations on $\mathcal{C}^\infty(M)$	$X_p : \mathcal{C}^\infty(M)$	$\text{Der}_p(\mathcal{C}^\infty(M))$		$X : \mathcal{C}^\infty(M)$	$\text{Der}(\mathcal{C}^\infty(M))$	
$\mathcal{C}^\infty(M)$ -multiplication	αX_p	$\mathbb{R}$ -vector space	$\mathbb{R}$ -vector bundle	$\alpha(p)X_p$	$\mathcal{C}^\infty(M)$ -module	
product of derivations	$X_p Y_p$ is not a vector			XY is not a vector field		
pushforward	$F_p^*(X_p) := \mathfrak{A}$	F_p^* is a linear map	F^* is a bundle map	when F is a diffeomorphism	F^* is a Lie algebra map?	
functor		$\text{Man}_* \rightarrow \text{Vect}$ $(M, p) \mapsto T_p$	$\text{Man} \rightarrow \text{Vect}$ $M \mapsto TM$			man to sheafs?!
commutator	-	-	-	$[X, Y] := XY - YX$ as endomorphism in $\mathcal{C}^\infty(M)$	$\text{Der}(\mathcal{C}^\infty(M))$ is a Lie $\mathbb{R}$ -algebra	sheaf to Lie $\mathbb{R}$ -algebras
flow of a vector field						
exponential	-	-	-	exponential of vector fields		
					By existence and uniqueness we have a global correspondence vector field	sheaf map
exponential and commutator				$\exp[tX, tY]$		
Lie derivative on $\mathcal{C}^\infty(M)$				$L_X f = Xf$		
Lie derivative on ΓTM				$L_X Y = [X, Y]$	$L_X : \Gamma TM \rightarrow \Gamma TM$ is a Lie algebra homomorphism?	
Lie derivative and pushes?						
					curves on $\Gamma TM \leftrightarrow$ time dependent vector fields	

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- Comparison of constructions with tangent vectors
- Tangent frames on smooth manifolds
- Tangent vector fields on a smooth manifold