Info

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Kleinian ultra-Schottky groups

$\leftrightarrow$

convex cocompact hyperbolic interior of 3-handlebodies

Definition. Kleinian Schottky groups

A **Kleinian ultra-Schottky group** is of the form

$$\Gamma = \langle g_1, \ldots, g_n \rangle \subseteq PSL(2)(\mathbb{C})$$

where there exits $2n$ disjoint simple closed curves $C_1, \ldots, C_n, C'_1, \ldots, C'_n \subseteq \mathbb{C}$ bound a domain D such that $C_m \overset{g_m}{\longmapsto} C'_m$ and $g_m(D) \cap D = \emptyset$ for each $1 \leq m \leq n$.
A *Klienian ultra-Schottky group* is **classical** if such disjoint simple closed curves C_k, C'_k can be chosen to be circles in $\mathbb{C}$.

> **Proposition:** A Kleinian ultra-Schottky group is free, geometrically finite with limit set $\Lambda(\Gamma) \neq \partial\mathbb{R}\boldsymbol{H}^3$ and is of purely hyperbolic-type. [1]

infinite Schottky groups

Definition. **A.2.** Now consider infinitely many pairs of disjoint circles $C_1, C'_1, \ldots$ as above. That is, the circles are pairwise disjoint, and no circle in the set separates ∞ from any other circle in the set. Choose elements f_m in $\mathbb{M}$, where f_m maps the outside of C_m onto the inside of C'_m. Let $G = \langle f_1, \ldots \rangle$. If G is discrete and if the circles all have a common exterior D, where D is a fundamental domain for G, then we call G an *infinite Schottky group*.
In the case that the circles do not all have a common exterior, that is, the closure of the union of the insides of all the circles is all of $\hat{\mathbb{C}}$, then we still call G an infinite Schottky group, provided it is discrete.

Current note has 1 direct children and 1 total descendants.

- stem
  - O+ R1 n
    - Discrete subgroups of $O^+(1,n)$ $\leftrightarrow$ hyperbolic n-manifolds/orbifolds
      - Schottky
        - Classical Schottky subgroups of $O^+(1,n)$
  - Objects stemming from $PSL(2)(\mathbb{C})$
    - Kleinian groups $\Gamma <_d PSL(2)(\mathbb{C})$ $\leftrightarrow$ Hyperbolic 3-manifolds/orbifolds $_\Gamma\backslash^{\mathbb{R}\boldsymbol{H}^3}$
      - Kleinian ultra-Schottky groups $\leftrightarrow$ convex cocompact hyperbolic interior of 3-handlebodies
        - Space of Schottky groups

And it has 7 siblings.

- stem
  - Objects stemming from $PSL(2)(\mathbb{C})$
    - Kleinian groups $\Gamma <_d PSL(2)(\mathbb{C})$ $\leftrightarrow$ Hyperbolic 3-manifolds/orbifolds $_\Gamma\backslash^{\mathbb{R}\boldsymbol{H}^3}$
      - Finitely generated, free Kleinian groups
      - 2-generated Kleinian groups
      - Finitely generated Kleinian groups $\leftrightarrow$ topologically tame oriented hyperbolic 3-manifolds/finite degree orbifold covers of
      - Building hyperbolic 3-manifolds by gluing polyhedra
      - Kleinian group with $\Lambda(\Gamma) \neq \partial\mathbb{R}\boldsymbol{H}^3$
      - Thick-think decomposition of hyperbolic 3-manifolds
      - Kleinian ultra-Schottky groups $\leftrightarrow$ convex cocompact hyperbolic interior of 3-handlebodies

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1. **H.2. Proposition.** *Let G be a Schottky group on the generators $g_1, \ldots, g_n$. Then*
(i) *G is a function group with no non-invariant components.*
(ii) *G is free on the n generators $g_1, \ldots, g_n$.*
(iii) *G is purely loxodromic.*
(iv) *D is a fundamental domain for G.*
(v) *G is geometrically finite.*
(vi) *The signature of G is (K, n) where K is an unmarked sphere.*
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